Sir David Brewster
1857
hotel de L' Europe

A

TREATISE

ON

NEW PHILOSOPHICAL

INSTRUMENTS.

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A

TREATISE

ON

NEW PHILOSOPHICAL

INSTRUMENTS,

FOR VARIOUS PURPOSES IN

THE ARTS AND SCIENCES.

WITH

EXPERIMENTS

ON

LIGHT AND COLOURS.

BY

DAVID BREWSTER, LL.D.

FELLOW OF THE ROYAL SOCIETY OF EDINBURGH, AND
OF THE SOCIETY OF THE ANTIQUARIES
OF SCOTLAND.

La revolution que nous demandons ne peut avoir lieu que dans les Instrumens; car si nous ne changeons point d'organes, nous verrons toujours la Nature comme nous l'avons vue.

BAILLY.

EDINBURGH:

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AND

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1813.

TO

JOHN PLAYFAIR, Esq.

PROFESSOR OF NATURAL PHILOSOPHY IN THE UNIVERSITY OF EDINBURGH, FELLOW OF THE ROYAL SOCIETY OF LONDON, AND SECRETARY TO THE ROYAL SOCIETY OF EDINBURGH.

In dedicating to you this volume, I discharge a duty, which both Friendship and Science impose upon me. The kind approbation which you formerly bestowed on some of the Instruments described in the following pages, was the strongest excitement to my earliest exertions: and the constant interest you have since taken in the progress of my researches, has often encouraged me to pursue them under circumstances not very favourable to physical investigations.

I shall ever regard it as the most fortunate event of my life, that I have shared so

much of your friendship, and enjoyed so many opportunities of admiring the highest moral and intellectual attainments: But, independently of these personal feelings, a scientific work would naturally shelter itself under a name which Science associates with her highest efforts.

DAVID BREWSTER.

EDINBURGH, *March* 15, 1813.

PREFACE.

In committing this volume to the public, I feel a degree of anxiety which is not usual with those who are accustomed to appear before its tribunal. Had it been a work of Science, the principal object of which was to detail the observations of preceding authors, or had I been able to command that uninterrupted leisure which is almost indispensible in experimental enquiries, this anxiety would have been considerably abated. But the subjects which I have had occasion to treat, are, in a great measure, new; and from the severity of my professional duties, I have been obliged to pursue them in the most irregular and interrupted manner, and under circumstances the most adverse to philosophical investigation. I rely, therefore, on

the indulgence of the reader for any defects which he may discover, and for any errors which may be ascribed to the circumstances under which I have written.

In the original plan of this work, I proposed to confine myself to the mere description of Instruments, and wished to leave the application of them to others who might afterwards investigate the subjects to which they referred. I was afraid, however, that they might thus be overlooked as untried inventions, which had not received the sanction of experience, and I therefore undertook a series of experiments upon Refractive Powers, by means of the Instrument which I had constructed for that purpose. From these experiments, I was naturally led to those upon Dispersive Powers,—a subject which presented a series of new and interesting results; and which is, perhaps, the most difficult within the whole range of experimental philosophy. With the aid of a new instrument, I have measured the dispersions of 137 substances, and by a series of calculations, which of

themselves would have filled a volume, I have determined the absolute dispersive powers of these various substances. The uncorrected colour which remains after equal and opposite dispersions, induced me to examine, with care, the action which different bodies exercise upon the differently coloured rays. The numerous experiments which were made with that view, while they establish this difference of action, and prove the existence of a *tertiary spectrum*, have suggested some maxims which may contribute to the improvement of the Achromatic Telescope.

The discovery of a new property impressed upon light, by transmission through the agate, opened a still wider and more alluring field of enquiry; and though this subject was not immediately connected with the description of any Instrument, I prosecuted it with renewed zeal, and examined the variations which light, thus modified, experienced from the action of refracting and reflecting substances. The power of

transparent bodies to destroy this property; the optical phenomena peculiar to mica and topaz; and the singular alternations of the prismatic colours which these bodies impress upon polarised light, were thus established by numerous experiments.

The leading results which were obtained in the course of these researches, may be thus enumerated.

1. It has been ascertained, that chromate of lead and realgar have a greater refractive power than the diamond, which has always been supposed to exceed every other body in its action upon light.

2. The chromate of lead possesses a double refraction, about thrice as great as that of Iceland spar.

3. The three simple inflammable substances have their refractive powers in the very order of their inflammability.

4. All doubly refracting crystals possess a double dispersive power, the greatest refraction being accompanied with the highest power of dispersion.

5. The fluates, viz. fluor spar and cryolite, have the lowest refractive powers of all solid substances, and the lowest dispersive powers of all bodies.

6. The agate, when cut by a plane at right angles to the laminæ of which it is composed, impresses upon a transmitted ray of light, the same character with one of the pencils formed by doubly refracting crystals.

7. This property of light, whether communicated by the agate, or by double refraction, or by reflection from transparent bodies, may be destroyed by transmitting the light, in one direction, through almost all mineral substances, and even through horn, tortoise shell, and gum arabic; while in another direction the original character of the ray is not altered. The axis of the substance in which the property is destroyed, I have called the *depolarising* axis; and the axis in which it is not altered, the *neutral* axis.

8. Mica and topaz, while they possess, in common with other bodies, the neutral and

depolarising axes, have also axes of a different kind. Each depolarising axis of the mica is accompanied with an *oblique neutral axis*, while the neutral axis, between the two common depolarising axes, has an *oblique depolarising axis*.

9. When the images of a luminous object are depolarised by the mica, they exhibit, by a gentle inclination of the plate, the most singular alternations of the prismatic colours. The same colours were observed in the topaz; and in a more perfect manner in a rhomboid of Iceland spar, which exhibited some new phenomena.

10. Light suffers a peculiar modification when reflected from the oxidated surface of polished steel, which seems to prove, that the oxide is a thin transparent film.

11. Light is partially polarised when reflected from polished metallic surfaces.

12. The light reflected from the clouds, the blue light of the sky, and the light which forms the rainbow, are all polarised.

13. It appears, from a great variety of

experiments, that bodies exert a different action upon the different coloured rays, oil of cassia having the least, and sulphuric acid the greatest, action upon green light.

14. The existence of a third, or a *tertiary spectrum*, has been established by numerous experiments; and a method has been pointed out, of employing this spectrum as a measure of the action which different bodies exercise upon the differently coloured rays.

Since this volume has been printed, an account of the labours of the French National Institute for 1812 has reached this country; and it appears from this abridged history of its proceedings, that M. Arago, of the Imperial Observatory, has noticed the alternations of colour produced by the transmission of polarised light through plates of mica, and that M. Biot has discovered the law of these remarkable phenomena. It appears, also, from some notices in the scientific journals, that M. Arago has discovered the depolarisation of light by transparent

bodies, and that Malus had ascertained before his death, that light was polarised by reflection from metallic surfaces.

These notices were unknown to me, till I had finished my experiments on the same subjects. The only memoir, indeed, which had reached this country, was one by Malus, on the polarisation of light by reflection from transparent substances; and neither I, nor any of my literary friends, had any means of knowing, that he had extended his experiments to polished metals, or that his associates in the National Institute had taken up the subject which he had so successfully begun.

After I had discovered the new property of the agate, I observed all the phenomena of depolarisation, and the peculiar properties of mica and topaz, in reference to light polarised in that particular manner. An account of my experiments was read at the meetings of the Royal Society of Edinburgh; and the experiments themselves were repeated at different times, and with

the most satisfactory results, before several of its members.

The discoveries of M. Arago, are almost exactly the same with those which are noticed in the 7th and 9th Articles of the preceding enumeration; but he does not seem to have discovered the oblique axis of depolarisation and neutrality, which I have found in mica and topaz, nor to have observed the singular phenomena which I have detected in Iceland spar. The generalisation of the facts alluded to in Article 9. which the celebrated Biot has the undivided merit of having accomplished, will probably throw a new light upon this perplexing branch of physical optics.

If this volume shall be favourably received by the Public, I shall take the first opportunity of laying before them a continuation of my experiments, and a description of several other Instruments, which I have been prevented from including in the present treatise.

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ERRATUM.

Page 269. line 12. For *concave* read *convex*.

TREATISE

ON

NEW PHILOSOPHICAL INSTRUMENTS.

BOOK I.

ON MICROMETERS.

The rapid advances which have been made in Astronomy during the two last centuries, have been almost entirely owing to the invention of two simple instruments, the Telescope and the Micrometer. When we compare the state of the science before the time of Galileo, who first applied the telescope to the examination of the heavens, with that more perfect system which is embraced by modern astronomers, we are astonished at the progress of discovery, and at the wonderful change which a single instrument has been able to effect. The discovery of five primary and

seventeen secondary planets; the determination of the figures and revolutions of several of the celestial bodies; the transits of the inferior planets over the sun; the successive propagation of light; the aberration of the celestial bodies; and the structure of the starry heavens; are a few of the additions which Astronomy has received from the science of Optics. In the prosecution of these discoveries, the micrometer became a powerful auxiliary to the telescope. It enabled the practical astronomer to measure, with accuracy, the smallest spaces in the heavens, to determine the diameters of the planets, and to ascertain those minute changes in the form and position of the celestial bodies, which have led to the most important information respecting the œconomy of the heavens. When we contrast the rude measurements of Kepler and Tycho with the delicate observations of Bradley, Maskelyne, Herschel, and Schroeter, we cannot fail to perceive the advantages of the micrometer, and the effect which any further improvement upon it must ultimately have, in contributing to the progress of the most perfect of the sciences.

In the present advanced state of Astronomy, every thing depends upon the accuracy of observation. The quantities to be ascertained are fre-

quently so very minute, as scarcely to become sensible in a series of years; and it is upon the precision with which these quantities are determined, that the speculations of the physical astronomer must depend. Had the ancients surveyed the heavens with the same instruments as the moderns, we should, at this time, have been intimately acquainted with the mechanism of the universe; we might have demonstrated those sublime views which Dr Herschel has lately unfolded respecting the motion of the solar system in absolute space; and by detecting the velocity with which it advances, and the direction in which it moves, we might have been able to trace a portion of its orbit round the centre of some greater system.

But it is not merely to the excellence of instruments, that the astronomer must confide the progress of his science. They must be simple as well as accurate, and should be so easily procured, as to be in the possession of all who have the inclination and the ability to use them. In examining the history of the other physical sciences, we find that much has been owing to the private exertions of men of science, and that from very slender means the most brilliant discoveries have arisen. Whereas, in practical astronomy, every thing has been done in a few national observato-

ries, and in those connected with universities, or in the possession of wealthy individuals. And yet, how rapid has been the progress of discovery! and how much more rapid would it have been, had every private astronomer enjoyed the means of observation! The science would now have possessed a code of facts, and would have presented a richer field for the speculations of the mechanical philosopher.

But it is not for the purposes of astronomy, merely, that the micrometer is a powerful instrument in the hands of the philosopher. It is of indispensible use in every branch of experimental science, where small portions of space are to be measured;—in the researches of the naturalist, where the size of minute objects, and the changes which they suffer, are to be ascertained; and in every case where magnitude and distance require to be measured. Hence the micrometer is a valuable instrument in the arts, as well as in the sciences; in trigonometrical surveys; in the practice of navigation; and in all military and naval operations.

Before we proceed to describe the micrometers which form the subject of this Book, it will be necessary, for the information of the reader, to give a general account of the most approved micrometer of the common construction, that he may be

able to estimate more correctly the simplicity and advantages of the instruments which are to come under his review.

The wire micrometer is an instrument fitted to the eyepiece of a telescope, for the purpose of measuring the angles subtended by small spaces in the heavens, such as the diameters of the planets, &c. by comprehending these spaces between two delicate parallel fibres, which retain their parallelism while they are opened and shut by a mechanical contrivance. This instrument is represented in Plate I. Fig. 1. as attached, along with the eyepiece GF, to the tube of the telescope AA, by means of a screw round the circumference of the circular plate *d*, (Fig. 1. and 2). Within this circular plate is fixed a conical ring *e e*, Fig. 2. fastened to a second ring BB, (Fig. 1. and 2) by three small screws, the ends of which may be seen in Fig. 2. By turning the screw D, therefore, (Fig. 1. and 2.) which works in teeth cut upon the circumference *d*, the whole eyepiece, along with the micrometer MN, may be made to revolve upon the conical ring *e e* as a centre. The inclination of the micrometer to the horizon, is pointed out upon the flat surface of the ring *d*, which is divided into 360 degrees, &c. Into the ring BB, (Fig. 1. and 2.) is screwed the common

Huygenian eyepiece FG. The micrometer itself is contained in the box EE, Fig. 1. and 3, which moves freely through the square holes in the ring BB, that the observer may follow an object while it is passing quickly through the field of the telescope. To shew the interior construction of the instrument, we have represented it in Fig. 3. with one of its sides removed. The fork of brass *n n* is fixed to the largest of the two small screws on the right hand side of the figure, and on the other end of the screw is placed a nut L, united to the divided head N, and moving, with its female screw, upon the male screw already mentioned. By turning the nut L, the fork *n n* may be moved backwards and forwards in the directions NM, MN, without any lateral shake. Within the fork *n n* is placed a second fork *o o*, fixed to another male screw on the left hand side of the figure, which may, in like manner, be moved backwards and forwards by means of the nut M, fixed to the divided head P. Across the lower side of each fork is extended a delicate fibre, or silver wire, *e*, *e*, which partakes of the motion of the fork to which it belongs, and both these fibres are so fixed that they always continue parallel, whatever be the distance to which they are separated by the motion of the forks. The lower sides of the forks are

made in such a manner, that the wires can just pass each other without touching; and at the instant of their passage, they must coincide so completely as to appear one wire. When this takes place, the index should point to the zero of the scale *a* within the micrometer, each division of which is equal to the interval between any of the threads of the male screw, and is of course passed over by either of the wires during one complete revolution of the nut L or M. The graduated heads N and P are each divided into 100 parts, so that the distance between the wires can at any time be ascertained to the 100dth part of one of the divisions of the scale *a*. These divisions are generally the 50th of an inch each, so that every unit on the graduated head corresponds to the 5000th part of an inch. A third wire *b*, lying in the direction of the screws, bisects the other two wires at right angles, and is intended to point out the direction in which the angle is to be measured. The angle subtended by the wires, when separated to any distance, is found by counting the number of revolutions, and parts of a revolution, of the divided head N or M, which are necessary to make the one wire move up to the other, and coincide with it. The number of seconds passed over by any of the wires during one revolution of the head N or M, must

be ascertained by actual experiment, that is, by measuring a base, and observing the space comprehended between the wires at the end of that base, where they are separated by so many revolutions;—or by observing the time employed by an equatorial star in passing from the one wire to the other. When the angle is thus determined which corresponds to any given number of revolutions, it may be found by simple proportion for any other number of revolutions; and these results may be conveniently put down in the form of a Table, to prevent the necessity of future calculation.

Let it be required, for example, to measure the sun's diameter, and let us suppose that two revolutions of the divided head separate the wires to such a distance, that they subtend exactly an angle of 60 seconds. Having directed the telescope to the sun, turn round the micrometer by means of the screw D, till the lower limb of the sun S*s*, (Plate I. Fig. 4.) just passes along the lowest parallel wire CD. Then turn the nut which moves the other wire AB, till it is at such a distance from CD, that the upper limb of the sun just passes along it. The sun S*s* is now comprehended exactly between the wires, and the angle which they subtend is therefore a measure of his diameter. Let the wire AB be now moved towards CD till they co-

incide, and suppose that it requires 60 revolutions of the divided head, and 46 hundredths of a revolution, to produce this coincidence. Then, since two revolutions are equal to 60 seconds, 60.45 revolutions will be equal to 30′.235, or 30′ 13″.5, the diameter of the sun required. The diameter thus measured, is obviously perpendicular to the direction of the sun's motion; but when we wish to measure the diameter which is parallel to the line of his daily motion, we must guide his upper or lower limb along the wire which bisects the parallel wires, and then separate the wires till the one extremity of the diameter is in contact with the first wire, at the very instant that the other extremity is in contact with the second wire. The motion of the sun, however, renders this observation so extremely difficult, that it is almost impossible to make it with any degree of accuracy.

By attending to the principles upon which this instrument is constructed, it will be easy to discover the numerous sources of error to which it is liable. The difficulty of finding the real zero of the scale, or the instant when the two wires appear to be in contact;—the error arising from the want of parallelism in the wires, or from a lateral shake in the forks which carry them;—the inflexion of light which takes place when the wires are

near each other; the complicated structure of the instrument; the minuteness of the scale, and of all it parts, but especially the difficulty of procuring screws in which the distance of the threads is always the same, are objections inseparable from the construction of this instrument. How far these sources of error are removed in some of the new instruments which we are now to describe, and what advantages they derive from simplicity of construction, must be determined by an attentive perusal of the following Chapters.

CHAP. I.

Description of a New Wire Micrometer.

THE diameter of the sun, or any portion of space, may be comprehended between a pair of wires placed in the eye-piece of a telescope, either by a mechanical or an optical contrivance; in the one case, by varying the distance of the wires till they contain exactly the solar disc; and in the other, by expanding or contracting the image of the sun till it exactly fills the space between a pair of fixed wires. Thus let S′s′, Plate I. Fig. 4. be the sun in contact with the lower wire CD, the wire AB may be moved into the position *ab*, so as to touch the upper limb S′ of the sun; or if the wires AB, CD, are both fixed, we may, by increasing the magnifying power of the telescope, expand the image S′s′ into S*s*, till its north and south limbs are in accurate contact with the fixed wires. In the first of these methods, which has already been explained in the description of the common wire micrometer, the

angle subtended by the sun is measured by the revolutions of the screw, which are necessary to bring the wire AB from a state of coincidence with CD into the position ab:—In the second method, which is the principle on which the new instrument is founded, the angle is measured by the change of magnifying power which is required to enlarge the solar image, till its diameter is exactly equal to the distance between the wires. Though the wires are in this case absolutely fixed, yet the angle which they subtend at the observer's eye continually changes with the magnifying power of the telescope: for if the sun $S's'$ fills half the space between the wires AB, CD, before the magnifying power is increased, the angle subtended by these wires must be equal to twice the diameter of the sun, or about 62 minutes; and when the solar image has been expanded to Ss, the wires AB, CD, only subtend an angle equal to the sun's diameter, or about 31 minutes; so that if this expansion of the sun's image has been produced by a gradual change in the magnifying power of the telescope, the wires must have subtended every possible angle between 31 and 62 minutes.

The gradual variation of the magnifying power, which is thus essential to the construction of the instrument, may be effected by different contrivan-

ces,—by changing the distance between the two parts of the achromatic eyepiece;* by separating one or more of the lenses of the compound object glass; or by making a convex, a concave, or a meniscus lens move along the axis of the telescope, between the object glass and its principal focus.

The last of these contrivances, which is, for many reasons, preferable to any of the other two, is represented in Plate I. Fig. 5. where O is the object glass, whose principal focus is at *f*, and L the separate lens, which is moveable between O and *f*. The parallel rays R, R, converging to *f*, after refraction by the object glass O, are intercepted by the lens L, and made to converge to a point F, where they form an image of the object from which they proceed. The focal distance of the object glass O has therefore been diminished by the interposition of the lens L, and consequently the magnifying power of the telescope, and the angle subtended by a pair of fixed wires in the eyepiece, have suffered a corresponding change. When the lens is at *l*, in contact with the object glass, the focus of parallel rays will be about φ; the magnifying power will be the least possible, and the angle of the wires will be a *maximum;* and when

* This contrivance will be described in Chap. VII. p. 59.

the lens is at l', so that its distance from O is equal to Of, the focus of parallel rays will be at f;—the magnifying power will be the greatest possible, and the angle of the wires will be a *minimum*. When the lens L has any intermediate position between l and l', the magnifying power and the angle of the wires have an intermediate value, which depends upon the distance of the lens from the object glass. Hence it appears, that the scale which measures these variations in the angle of the wires, may always be equal to the focal length of the object glass; and it may be shewn in the following manner, that it is a scale of equal parts, the changes upon the angle being always proportional to the variation in the position of the moveable lens.

The point f being that to which the rays incident upon L always converge, we shall have by the principles of optics,

$$F+Lf : F = Lf : LF$$

F being equal to the focal length of the lens L. Now it is obvious, that the magnitude of the image formed at F, after refraction through both the lenses, will be to the magnitude of the image formed at f by the object glass O, (or by both lenses when L is at l',) as LF is to Lf; for the image formed at f is the virtual object from which

the image at F is formed, and the magnitude of the image is always to the magnitude of the object directly as their respective distances from the lens. Hence the magnifying power of the telescope when the lens L is in these two positions, is in the ratio of LF to Lf, consequently the angle subtended by the wires, which must always be inversely as the magnifying power, will be as Lf to LF.

By making $Lf=b$, the preceding formula becomes

$$F+b : F=b : LF. \quad \text{Hence}$$

$$LF=\frac{Fb}{F+b}$$

Then calling A the least angle subtended by the wires, or the angle which they subtend when the lens L is at l', and α the angle which they subtend when the lens is at L or in any other position, we have

$$A : \alpha = LF : Lf, \text{ that is}$$

$$A : \alpha = \frac{Fb}{F+b} : b, \text{ and}$$

$$\alpha = A+\frac{Ab}{F} = \text{the ANGLE for any distance } b.$$

Calling P the greatest magnifying power, and π the magnifying power for any distance b, we shall have

$$P : \pi = b : \frac{Fb}{F+b}, \text{ and}$$

$$\pi = \frac{PF}{F+b} = \text{the POWER for any distance } b.$$

Making A=20, P=20, F=10, and b=0, 1, 2, 3, 4, successively, we obtain from these two formulæ the results in the following Table.

Different values of b.	*Calculated magnifying powers.*	*Differences.*	*Calculated angles.*	*Differences.*
0	20.00000		20′	
		1.81818		2
1	18.18182		22	
		1.51515		2
2	16.66666		24	
		1.28205		2
3	15.38461		26	
		1.09890		2
4	14.28571		28	

Hence it appears, that when the different values of b are in arithmetical progression, the angle α of the wires varies at the same rate, and therefore the scale which measures these angular variations is a scale of equal parts. The magnifying power, however, does not vary with equal differences, and consequently a scale for measuring its variations, if any scale were wanted, is not a scale of equal parts.

Having thus ascertained the nature of the scale, we shall now proceed to point out the method of constructing it. It is obvious that the length of

the scale is arbitrary, and may be made equal either to the whole focal length Of of the object glass, or to any portion of it. If the lens L moves along the *whole* length of the axis Of, the angle subtended by the wires can be varied to a greater degree than if the lens moves only along a portion of the axis; but as this advantage may be obtained by a contrivance hereafter to be described, it will be found more convenient for astronomical purposes to make the lens moveable only along a part of the axis, as from L towards f.

Let us suppose, therefore, that when the object glass O is 36 inches in focal length, 10 inches will be a convenient length for the scale, and that the telescope is constructed so that the lens L can move freely through that space reckoned from f, the next thing to be determined is the focal length of the lens L. It is evident that a lens of 6 inches focal length will produce a much greater diminution of magnifying power, and consequently a much greater increase upon the angle of the wires in moving from f to L than a lens of greater focal length; so that the value of the whole scale in minutes or seconds, or the increase in the angle occasioned by the motion of the lens from f to L, must be inversely as the focal length of the moveable lens. If the ängle of the wires is 26 minutes, for

example, and if the magnifying power of the telescope is diminished from 40 to 30 by the motion of the lens from f to L; then when the lens is at L, the angle of the wires will be 34′ 40″, for

$$30 : 40 = 26' : 34'\ 40''.$$

Hence we have a scale of 10 inches to measure 26′—34′ 40″, or 8′ 40″, and therefore every tenth of an inch on the scale will be equal to 5″.2.

If we employ a lens of much greater focal length, so as to diminish the magnifying power only from 40 to 35, and if the angle of the wires is 29 minutes; then when the lens is at L, the angle of the wires will be 33′ 9″ nearly, for

$$35 : 40 = 29' : 33'\ 9''.$$

And hence we have a scale of 10 inches to measure 29′—33′ 9″, or 4′ 9″, consequently every tenth of an inch on the scale corresponds to 3″.3. From this it will be manifest, that the accuracy of the scale is increased by increasing the focal length of the moveable lens.

The two preceding examples are suited to a micrometer for measuring the diameters of the sun and moon at their various distances from the earth; but, in order to shew the resources of the principle on which the instrument is founded, we shall take another example, better adapted to this purpose.

Let us suppose that the pair of fixed wires subtends only an angle of 40″, for the purpose of measuring the distance between double stars, or the diameters of some of the smaller planets, that the telescope magnifies 300 times, and that the lens L in its motion from f to L, through a space of 10 inches, diminishes the power of the instrument to 240; then when the lens is at L, the angle of the wires will be 50″, for

$$240 : 300 = 40'' : 50''.$$

Hence we have a scale of 10 inches to measure 40″—50″, or 10″, so that every inch of the scale corresponds to 1″, and every 10th of an inch to 6‴. From this it follows, that the accuracy of the scale increases as the angle subtended by the fixed wires diminishes.

If it should be found convenient to make each division of the scale correspond to a greater variation in the angle than in any of the examples which we have given, it would then be proper to make use of a vernier for subdividing the units of the scale.

In order to shew more clearly the method of completing the scale, we have represented a telescope furnished with a micrometer, in Plate I. Fig. 6, where AB is the principal tube, with the object glass at B; CD a secondary tube, at the right

hand extremity of which is fixed the lens L, (Fig. 5.) which is moved backwards and forwards with the tube, by the milled head F; and E the eye-piece, which is adjusted to distinct vision by the milled head G. The small index i projecting from the principal tube below A, and furnished with a vernier scale if necessary, points out the divisions on the scale. Let it now be required to construct the scale for the 2d Example, where the lens L, by moving from f to L, changes the power of the telescope from 40 to 35. Having moved the tube CD as far out as possible, by the milled head F, mark the point of it at n to which the index i points, and this will be the beginning or zero of the scale. Adjust the eye tube E to distinct vision, and find by experiment* the angle subtended by the fixed wires: Let this angle be 29 mi-

* In order to find the angle subtended by a pair of wires AB, CD, Plate I. Fig. 4, direct the telescope, the object glass of which is supposed to be at the point A, Plate II. Fig. 2, to any upright object MN, with a plain surface, perpendicular to the axis of the telescope, and placed at a convenient distance, 500 feet for example, and observe the space which the wires appear to occupy, or the points B, C, which the wires seem to cover, taking particular care that the line joining these points is perpendicular to the wires. Let the space BC be 4 feet 2.57 inches. Bisect BC in D, and draw DA; then in the right angled triangle ADB, we have AD=500 feet, and BD=2 feet 1.285 inches, to determine the

nutes. Move the tube CD as far in as possible by means of the nut F, till the index i points to m, and mark this as the other extremity of the scale. Let the eye tube E be again adjusted to distinct vision, and the angle subtended by the wires again determined experimentally; and let this angle be now 33′ 9″. In order to find the point of the scale

angle BAD, which, by the simplest case of plain trigonometry, will be found to be 14′ 30″, so that the whole angle BAC, or that subtended by the wires, will be 29′.

It is obvious, however, that on account of the proximity of the object MN, the image of it in the telescope is formed by rays which fall diverging upon the object glass O, and therefore this image will be formed at f', Plate I. Fig. 5, at a greater distance than the principal focus f. Hence the magnifying power will be greater, and the angle of the wires less than they would have been had the object MN been infinitely distant. It is necessary, therefore, to find the corrected angle BAC, so that we may have the real value of that angle when the telescope is directed to the heavenly bodies. Let Of, the focal length of O, Plate I. Fig. 5. be called φ; D the distance of the object MN; a the angle found by experiment; x the correct angle; F the focal length of L; and b the distance of the lens L from the focus f. Then we have, by the principles of Optics, $\frac{\varphi^2}{D-\varphi}=ff'$, the increase of focal length. Calling this value of ff' m, we have $b+m=Lf'$, and, by the principles of Optics, $F+b+m : F=b+m : \frac{Fb+Fm}{F+b+m}$ for the new value of LF or for LF′. Hence LF : LF′$=a : x$; that is, $\frac{Fb}{F+b} : \frac{Fb+Fm}{F+b+m}=a : x$; a formula from which the corrected angle x may be readily found.

corresponding to 33′, say, As 4′ 9″, the value of the whole scale, is to 10 inches, the length of the scale, so is 9″ to 36 hundredths of an inch, which being set from *m* to *o*, will mark out the point *o* as corresponding to 33′. The space *o n* being divided into four parts for minutes, and each minute into as many divisions as possible, the micrometer will be ready for use. If great accuracy is required, every unit of the scale might be determined experimentally, by any of the methods mentioned in the preceding note.

The instrument thus constructed, is capable of measuring angles only between 29′ and 33′ 9″, and is therefore peculiarly fitted for determining the diameters of the sun and moon. Its range, however, could easily have been extended, by lengthening the tube CD, or by employing a moveable lens, of smaller focal length;—or instead of one pair of wires, we might use several pairs, as AB, CD, *ab*, *cd*, $\alpha\beta$, $\gamma\delta$, Plate II. Fig. 1. so placed that only one pair should be in the field of view at a time, and that the least angle of the second

Instead of using the preceding trigonometrical method, the angle subtended by the wires may be more easily found by observing the time in which an equatorial star passes from the one wire to the other. This portion of time converted into minutes, at the rate of 4 minutes to a degree, will be the angle subtended by the wires.

pair should be equal to the greatest angle of the first pair, and the least angle of the third pair equal to the greatest angle of the second.

When the micrometer is constructed on these principles, it is certainly free from almost all those sources of error with which the wire micrometer is affected. The imperfections of the screw, the errors arising from the uncertainty of the zero, from the bad centering of the lenses, from the want of parallelism in the wires, and from the minuteness of the scale, are completely removed. Nay, if the scale is formed by direct experiments, whatever errors may exist in the instrument, are actually corrected; for as the sources from which these errors proceed existed in the instrument during the formation of the scale, they cannot possibly affect the result of any observation. The scale is in fact the record of a series of experimental results, and the observation must be as free from error as the experiments by which the scale was formed. It would, therefore, be of great advantage, in micrometrical observations, to make the points B, C, Plate II. Fig. 2, with which the wires appear to come in contact, as luminous as the objects to which it is intended to apply the instrument, or rather to have a series of results for objects of various degrees of illumination.

The application of this micrometer to a telescope for measuring distances, will be explained in Book III. Chap. I., to which we must also refer the reader for the discussion of several points connected with the theory of the instrument.

CHAP. II.

Description of a New Micrometer for Reflecting Telescopes.

In applying to the reflecting telescopes of Gregory and Cassegrain the principle which has been explained in the preceding Chapter, we are led to the formation of a micrometer, remarkable for the simplicity of its construction; and what, at first sight, may appear paradoxical, we may convert a Gregorian or a Cassegrainian telescope into a very accurate micrometer, almost without the aid of any additional apparatus.

It will be readily seen by those who understand the theory of these telescopes, that their magnifying power may be increased merely by varying the distance between the eyepiece and the great speculum; and then producing distinct vision by a new adjustment of the small mirror. Hence a pair of wires fixed in the eyepiece may be made to subtend different angles, solely by having that

part of the instrument moveable along a portion of the common axis of the two mirrors.

In order to understand this, let SS, Plate II. Fig. 3. be the great speculum of a Gregorian telescope, having a round hole in its centre, and placed at the extremity of the tube AA; and let M be the small speculum, whose focus is G, and centre H, attached to an arm MQ, and moveable along the axis of the instrument by means of a screw and milled head. The rays RR, proceeding from the lower part of any object, and falling upon the speculum SS, will be reflected to R′, and will there form an image of that part of the object. In like manner, the rays *rr* will form an image of the upper part of the object at *r*′. The rays diverging from the image R′*r*′, and intercepted by the small speculum M, will form another image R″*r*″, at the distance MF; which being viewed by the eyeglass at E, whose focal distance is FE, will appear distinct and magnified to the observer.

Let us now suppose that the lens E, or the eyepiece of the telescope, (which is generally a Huygenian eyepiece, with two glasses,) is moved by a suitable apparatus into the position E′, and that a point F′ is taken, so that F′E′ may be equal to FE. Then it is manifest, that, in order to have a distinct view of the object in this new position of the

eyepiece, the image formed by the small speculum must be brought to F' in the focus of the lens E'. But as the place of the first image $R'r'$ is in no respects changed by the change of position in the eyepiece, the formation of the image at F can be effected only by bringing the small mirror M into a position M', nearer the image $R'r'$ than it was before; and as the space MM' through which it has been moved, in order to converge the rays to F', must necessarily be less than $FF'=EE'$, the space through which the eyeglass has moved; the distance $M'F'$ of the new image at F' from the small mirror, must be greater than MF, the distance of the other image at F, in the ratio of $M'F'$ to MF; and the magnifying power of the instrument must at the same time be increased, and the angle subtended by the wires diminished.

Let us now see to what this variation of power amounts, and what must be the nature of the scale by which the changes of the angle subtended by the fixed wires ought to be measured. By the principles of Catoptrics, we have

$$Mf : MF = fG : GH, \text{ consequently}$$

$$MF = \frac{Mf \times GH}{fG}$$

Now $MF' = MF + EE$

$$M'F' = MF + EE - MM', \text{ and}$$

$$M'f = Mf - MM'$$

When the small mirror, therefore, is at M′, we shall have

$$\mathrm{M}f-\mathrm{MM}' : \mathrm{MF}+\mathrm{EE}'-\mathrm{MM}'=f\mathrm{G} : \mathrm{GH}, \text{ and}$$

making $\mathrm{MM}'=x$, $f\mathrm{G}=p$, $\mathrm{M}f=m$

$\mathrm{EE}'=a$, $\mathrm{GH}=q$, $\mathrm{MF}=n$, we obtain

$$x=\frac{pn+pa-mq}{p-q}.$$

The magnifying power produced by the small mirror when at M is evidently equal to $\frac{\mathrm{MF}}{\mathrm{M}f}$ or $\frac{n}{m}$, and the magnifying power produced by the small mirror when at M′ is equal to $\frac{\mathrm{M'F'}}{\mathrm{M'}f}$ or $\frac{n+a-x}{m-x}$, so that if we call π, the least magnifying power of the telescope, and P, any other magnifying power, we have

$$\pi : \mathrm{P}=\frac{n}{m} : \frac{n+a-x}{m-x}, \text{ and}$$

$$\mathrm{P}=\frac{\pi m}{n}\times\frac{n+a-x}{m-x}.$$

If A be the greatest angle of the wires, and α any other angle, then since the angles are inversely as the magnifying powers, we have

$$\mathrm{A} : \alpha=\frac{n+a-x}{m-x} : \frac{n}{m}, \text{ and}$$

$$\alpha=\frac{\mathrm{A}n}{m}\times\frac{m-x}{n+a-x}.$$

Let us now suppose $a = 0, 1, 2, 3, 4$, successively,

$$p = 1,$$
$$q = 6,$$
$$\pi = 20,$$
$$A = 20,$$

then since $m = p + q$, we have

$$m = 7,$$

and since $n = \frac{m \times q}{p}$, we have

$$n = 42.$$

Then, by the different formulæ, we obtain the following results:

Values of a.	Values of x.	Calculated magnifying powers or values of π.	Differences.	Calculated angles or values of α.	Differences.	Angles if the scale were of equal parts.
0	0	20.000		20.000		20.000
1	.2	20.980	980	19.065	935	19.115
2	.4	22.022	1042	18.165	901	18.230
3	.6	23.125	1103	17.297	868	17.345
4	.8	24.301	1176	16.460	837	16.460

From these results it appears, that when a varies in arithmetical progression, the values of α, or the calculated angles, do not vary in the same rate, and consequently the scale which measures these angular variations is not a scale of equal parts, though, in the case for which the Table is calculated. the deviation is not very considerable.

In the formation of this micrometer, we may either construct the scale from calculation, after the two extreme points of it have been fixed experimentally, by the method already described in the preceding Chapter; or all the points of the scale may be determined by direct experiment. It would perhaps be more convenient to divide the scale into equal parts, and to construct a Table from the preceding formulæ, for the purpose of shewing, by inspection, the angle which corresponds to any number of these equal divisions.

CHAP. III.

Description of a New Divided Object Glass Micrometer.

THE common divided object glass micrometer, was invented about the same time by Savery and Bouguer, and was afterwards greatly improved by our countryman Mr Dollond. It consists of two semilenses, A, B, Plate II. Fig. 4. of the same focal length, formed by dividing a convex lens into two equal parts, by a plane which passes through its axis. The centres of these semilenses are made to separate, and to approach each other, by means of a screw or pinion along the line AB, and the distance of their centres is measured upon a scale subdivided by a vernier, or, as in the wire micrometer, by a graduated head fixed upon the screw.

If it is required to measure the angle subtended by two objects, MN, the semilenses are separated till the two images of these objects are

in contact, or till the image of M, formed by the semilens A, appears to be in contact with the image of N, formed by the semilens B. When this happens, the angle subtended by the objects is equal to the angle subtended by AB, the distance of the centres of the semilenses at the point F, or the focus of the lenses where the contact of the images takes place. It is manifest, that an image of M will be formed in the line A F, and at F the focus of rays diverging from M. In like manner, an image of N will be formed in the line B F, and at F the focus of rays proceeding from the radiant point N. Hence it is obvious that the angle subtended at F by M N, is the same as the angle subtended by AB at F. The angle AFB may be easily found trigonometrically, the sides AB and OF being known; but, as this angle is generally very small, it may, without any perceptible error, be considered as proportional to the subtense AB, or the distance between the centres of the semilenses. By determining, therefore, experimentally, the angle which corresponds to any distance AB of the semilenses, we may, by simple proportion, find the angle for any other distance.*

*A complete perspective view, and a description of the most

With this preliminary information, it will not be difficult to understand the theory and construction of the New Divided Object Glass Micrometer. This instrument consists of an achromatic object glass, LL, (Plate II. Fig. 5.), having two semilenses, A, B, represented in Fig. 6., moveable between it and its principal focus *f*. These semilenses are completely fixed, so that their centres are invariably at the same distance; but the angle subtended by the two images which they form, is varied by giving them a motion along the axis O *f* of the lens LL. When the semilenses are close to LL, the two images are much separated, and form a great angle; but, as the lenses are moved towards *f*, the centres of the images gradually approach each other, and the angle which they form is constantly increasing. By ascertaining, therefore, experimentally, the angle formed by the centres of the images, when the semilenses are placed close to LL, and also the angle which they subtend when the semilenses are at *f*, the other extremity of the scale, we have an instrument which will measure, with the utmost accuracy, all intermediate angles.

approved Divided Object Glass Micrometer, of the common construction, will be found in the EDINBURGH ENCYCLOPÆDIA, Art, *Astronomy*, vol. II. p. 734.

In constructing this micrometer for astronomical purposes, the semilenses may be made to move only along a portion of the axis O *f*, particularly if the instrument is intended to measure the diameters of the sun and moon, or any series of angles within given limits. By increasing the focal length of the semilenses, or by diminishing the distance between their centres, the angles may be made to vary with any degree of slowness, and of course each unit of the scale will correspond to a very small portion of the whole angle. The accuracy and magnitude of the scale, indeed, may be increased without limit; but it is completely unnecessary to carry this any farther than till the error of the scale is less than the probable error of observation.

Let us now examine the theory of this micrometer, and endeavour to ascertain the nature of the scale for measuring the variations of the angle. For this purpose, let LL, Plate II. Fig. 5. be the object glass which forms an inverted image, *m n*, of the object MN, and let the semilenses AB, having their centres at an invariable distance, be interposed between the object glass and its principal focus, in such a manner, that their centres are equidistant from the axis O *f*. Now, it is obvious, that the size of the image *m n*, is propor-

tional to the size of the object MN; and, as the angle subtended by MN depends upon its size, the magnitude of the image *m n* may, in the case of small angles, be assumed as a measure of the angle subtended by MN. As the rays which proceed from the point M, are all converged to *m* by means of the lens LL alone, the ray *b* A, which passes through the centre of the semilens A, must of course have the direction *b m*; and, as it suffers no refraction in passing through the centre of A, it will proceed in the same direction *b* A *m*, after emerging from the semilens, and will cross the axis at F. For the same reasons, the ray *c* B, proceeding from N, and passing through the centre of B, will cross the axis at F, as it advances to *n*. If the distance of F from A and B happens to be equal to the focal length of the lenses A and B, when combined with LL, distinct images of M and N will be formed at F, and they will appear to touch one another; and the line *m n* being the size of the image that would have been formed by the lens LL alone, will be a measure of the angle subtended by the points MN. If the point F, where the lines A *m*, B *n* cross the axis, should not happen to coincide with the focus of the lenses A, B, when combined with LL, then let this focus be

at F′, nearer A and B than F. Draw the lines A F′ m', B F′ n', then it is obvious, that if the angle subtended by MN were enlarged, so as to be represented by n' m', instead of n m, or so that the lens LL alone would form an image of it equal to n' m', the point of intersection F would coincide with the focus F; so that, in every position of the lenses A, B, with respect to LL, the points MN may always be made to subtend such an angle, that when they are placed before the telescope, the points F, F′ will coincide, and consequently the images of the points M, N will be distinctly formed at F′, and will be in contact. Whenever this happens, the space n' m' will be a measure of the angle thus subtended by MN. Hence it follows, that whatever be the position of the semilenses A, B, on the axis O f, the rays b A, c B, which pass through the centres of the semilenses, will cross the axis at some point F, corresponding with the focus of rays diverging from M, N, and will mark out the size of the image n' m', and consequently the relative magnitude of the angle subtended by the two points M, N.

From the equality of the vertical angles A F′ B, n' F′ m', and the parallelism of the lines AB, n' m', we shall have

$$n'\,m' : AB = f\,F' : GF';$$

and calling $f\,F' = b$, as in page 15, and considering that $GF' = \frac{F\,b}{F+b}$, F being the focal length of the semilenses, we have

$$n'\,m' : AB = b : \frac{F\,b}{F+b}, \text{ and consequently}$$

$$n'\,m' = AB + \frac{AB \times b}{F}.$$

Now, calling AB = 2, F = 10, and b = 1, 2, 3, successively, we shall obtain

$$n'\,m' = 2 + \frac{2 \times 1}{10} = 2.2$$

$$n'\,m' = 2 + \frac{2 \times 2}{10} = 2.4$$

$$n'\,m' = 2 + \frac{2 \times 3}{10} = 2.6$$

from which it appears, that when b is in arithmetical progression, the angle $n'\,m'$ varies at the same rate, and consequently the scale which measures the variations of the angle, subtended by the centres of the two images, is a scale of equal parts.

This instrument undergoes a very singular change, when constructed as in Plate II. Fig. 7., so that the semilenses are outermost and immoveable, while another lens, LL, is made to move along the axis G f. In this case, a double image is formed as before, but the angle subtended by the centres of the images never suffers any change

during the motion of the lens LL along the axis of the telescope. If the two images are in contact when the lens LL is close to the semilenses, they will continue in contact in every other position of LL; but the magnitude of the images is constantly increasing during the motion of LL towards *f*, the principal focus of the semilenses. The reason of this remarkable property will be understood from Fig. 7., where M, N, are two objects placed at such an angle, that the rays passing through the centres A, B, of the semilenses, cross the axis at F, the focus of the combined lenses for rays divergent from M and N. In this case, distinct images of M and N will be formed at F, and will consequently be in contact. If the lens LL is removed to the position L′ L′, the rays M *m*, N *n*, which are incident upon it at the points *m* and *n*, having the same degree of convergency as before, will be refracted to F′, the focus of the combined lenses for rays diverging from MN. Two distinct images of the object will therefore be formed at F′, and these images will still be in contact. In like manner, it may be shewn, that whatever be the position of the lens LL between G and *f*, the rays M *f*, N *f*, will cross the axis at a point coincident with the focus of the combined lenses, and

will there form two images always in contact. Hence it follows, that though the magnifying power of the instrument is constantly changing with the position of the lens LL, yet the angle subtended by the centres of the two images never suffers the least variation.

The application of the Divided Object Glass Micrometer to a telescope for measuring distances, and to a coming-up glass for ascertaining whether a ship is approaching to, or receding from, the observer, will be described in Book III. Chap. II., to which we have reserved the discussion of several points connected with the theory of the Instrument.

CHAP. IV.

Description of a Luminous Image Micrometer.

THE object of this instrument is to measure the angle subtended by two luminous points; and it may be applied with considerable advantage in ascertaining the distances between fixed stars that are comprehended in the field of a telescope;—in determining the inclination which a line joining the centres of two stars forms with the direction of their apparent motion;—in surveying or in measuring distances during night;—and in finding the position of a ship, with respect to fixed lights placed upon the coast.

Let LL (Plate III. Fig.1.) be the object glass of an achromatic telescope, whose focus is f, and M, N, two luminous points, images of which will be formed at m and n. When the images m, n are in the anterior focus f, of the eyeglass E, they will appear distinct and luminous points; but when

the eyeglass is pushed forward to E′, so that f' is its anterior focus, the images m, n, instead of being points as before, will be sections of the cone of rays LL m, LL n, or circular, and well defined images of light, whose diameters are $a\,b$, $c\,d$. When the lens E is brought still farther forward to E″, so that its anterior focus is at f'', the diameters of these circular images will increase to $\alpha\,\beta$, $\gamma\,\delta$, and the angular distance of their centres will diminish. Hence, by advancing the eyeglass E, till its anterior focus is at φ, where the two circular images are evidently in contact, we obtain a measure of the angle subtended by the luminous points MN, in the same way as in the divided object glass micrometer. The contact of the images will always take place at the point φ, where the two cones of rays begin to separate; and, as this point is marked out by the intersection of the two extreme rays which form the interior sides of each cone, its distance from f, or φf, will not vary in the same ratio with $m\,n$, or the angle subtended by the luminous points MN.

In order, therefore, to find the nature of the scale, let L $\varphi\,m$, L $\varphi\,n$, be the extreme rays which form the interior sides of each cone, when $m\,n$ represents the distance of the images, or the angle subtended by MN, then,

$$mn : \mathrm{LL} = \varphi f : \varphi \mathrm{G}, \text{ and}$$

$mn = \frac{\mathrm{LL} \times \varphi f}{\varphi \mathrm{G}}$, that is, the angle will vary as $\frac{\varphi f}{\varphi \mathrm{G}}$. Hence, when the angle is infinitely small, φf will be equal to 0; and consequently the scale will commence at f, the focus of the lens LL. By determining, therefore, experimentally, the angle subtended by the luminous points, for any given value of φf, the scale may be easily constructed. Thus, let Gf, the focal length of LL, be 24 inches, and let it be found, from direct experiment, that the two circular images are in contact, when the angle subtended by the luminous points is 4 minutes, and when φf is 6 inches. In order to determine the angle when φf is 3 inches, we have $\varphi \mathrm{G} = 18$ inches when $\varphi f = 6$; and $\varphi \mathrm{G} = 21$ inches when $\varphi f = 3$. Hence,

$$\frac{6}{18} : \frac{3}{21} = mn : m'n', \text{ that is,}$$

$\frac{1}{3} : \frac{1}{7} = 4' : 1'\ 42''\ \frac{6}{7}$, consequently, the value of each unit of the scale may be accurately determined. It is manifest, that the same effect will be produced by pulling out the eyeglass E beyond the focal point f. The luminous points will expand into circular images as before, and these will come

into contact when the eyeglass E is pulled out to a proper distance from the focus; but this distance will always be greater than the corresponding distance within the focus at which the contact takes place.*

The scale, therefore, for measuring the angle of the luminous points, might stretch in both directions from the focus f; so that two measures of the angle will be obtained, one when the eyeglass is between the object glass and its focus f, and the other when it is pulled out beyond that focus. The medium between these observations will then be the angle required.

As the focal length Gf, not only varies with the distance of the object, but often suffers a considerable change from a variation of temperature and from other causes, the scale should be moveable; so that the index may always be at zero, when the luminous points appear distinct and well defined to the observer. The luminous points are now brought into the field of view, and the eyepiece E is made to approach the object glass, till the circular images are in contact. The index will then mark upon the scale the angular distance of the points. The observation

* See Book III. Chap. iii. p. 197.

may be repeated by pulling out the eyepiece beyond *f*, till the circular images again touch each other, and, if the scale does not give the same result, the medium of the two observations may be assumed as the true angular distance of the points.

In order to avoid the necessity of a moveable scale, the adjustment for a change of temperature, and for the aberration of focal length, may be effected by placing the object-glass in a small tube, which can be pushed out or in about two inches; so that if the focal length of the object-glass is increased, the focal point *f* may be kept opposite to the zero of the scale by pulling out the object glass tube; and, on the contrary, if its focal length is shortened, the focal point may be kept stationary by pushing in the object-glass.

It is a curious fact, that the circular images, or the sections of the cone of rays, are never so distinct and well defined after the rays have crossed at *m* and *n*, as they are before the rays have reached these points; and, therefore, it would be proper to place less confidence in the observations taken beyond *f*, and to make the scale commence opposite to *f*, and stretch towards the object-glass of the telescope. In

this case it is necessary to adjust the index to zero, when the luminous points are seen distinctly.

The only objection to which this micrometer seems to be liable is, that the expanded images will not be sufficiently defined in order to perceive their mutual contact. The discs, however, are much better defined than could have been imagined; and the distinctness of their termination must always increase with the goodness of the object glass.

The preceding method of creating a luminous disc may be employed for other purposes in Practical Astronomy. In finding the right ascension of the fixed stars, we might ascertain the arrival of the east and west sides of their disc at the vertical wires of the transit instrument, and obtain a very accurate result, by taking a mean of the two observations; and in the same manner, the observed altitudes of the north and south limbs would furnish us with a very exact measure of their declination.

The same principle also conducts us to an accurate method of performing one of the most delicate observations in Practical Astronomy, to measure the angle contained by a line joining the centre of two stars nearly in contact, and a line lying in the direction of their apparent motion,

This observation was first tried by Dr Herschel on double stars, for the purpose of ascertaining the angular motion of the smallest star round its companion. He placed two wires AB, CD, Plate III. Fig. 2. in the field of view, one of which, AB, had a motion of rotation, so as to form every possible angle with the other. The moveable wire AB was then placed in the direction of the apparent motion of the star S, and the other wire CD was made to form such an angle with it, that both the stars arrived at this wire at the same time. The inclination of the wires, which Dr Herschel calls the angle of position, gives us the angle formed by the line joining the centres of the stars S, *s*, and the direction of their apparent motion. In this way of making the observation, the want of a visible disc in the star renders it difficult to place one of the wires parallel to the equator; and there is a still greater difficulty in ascertaining the arrival of the largest star at the angular point, or the intersection of the two wires. By converting the images of the star, however, into luminous discs, as is represented in Fig. 3, we not only get rid of these difficulties, but introduce a new source of accuracy into the observation. Let the luminous discs, therefore, be expanded till the one encroaches upon the other, and let one of the

wires AB be so fixed that the southern limb of the lower disc may glide along its surface, it is then obvious that a line joining the two points *m n*, where the luminous circumferences intersect each other, is perpendicular to the line joining the centres of these discs, and will therefore form an angle COA with the fixed wire AB, equal to the complement of the angle SRO, which it is required to measure. By making the moveable wire, therefore, pass through the two points where the luminous circles intersect each other, the micrometer will shew the complement of the angle required.*

The application of this micrometer to a telescope for measuring distances, for surveying during night, and for determining the position of a ship with respect to lights placed upon the coast, will be explained in Book III. The principle of expanding a portion of light into a circular image, will be found of extensive use, and will be employed in a subsequent Chapter in the construction of a general Photometer.

* See Book II. chap. V.

CHAP. V.

Description of a Circular Mother-of-pearl Micrometer.

In the Philosophical Transactions for 1791, the late Mr Tiberius Cavallo has described an ingenious and simple micrometer, invented by himself, and excellently fitted for measuring small angles with accuracy and expedition. It consists of a slip of mother-of-pearl minutely subdivided, and stretched across the diaphragm that is placed in the anterior focus of the first eye-glass of an achromatic telescope. The angle subtended by any number of its divisions is then ascertained by experiment, and therefore the value of any other number can be found either by simple proportion, or from a table computed for the purpose.

This simple micrometer is very convenient in portable telescopes, where the eyepiece has a motion about its axis; but in telescopes supported

upon stands, where the eyepiece is moved by a rack and pinion, the slip of mother-of-pearl cannot turn round upon its axis, and, consequently, can only measure angles in one direction. This difficulty, indeed, might be surmounted by a mechanical contrivance for turning the diaphragm about its centre, or more simply, by giving a motion of rotation to the tube which contains the first and second eye-glasses. As such a change in the eyepiece, however, is often inconvenient and difficult to be made, Mr Cavallo's micrometer has this great disadvantage, that it cannot be used in reflecting telescopes, or in any achromatic telescope where the adjustment of the eyepiece is effected by rack-work, unless the structure of these instruments is altered for the purpose. Another disadvantage of this micrometer arises from the slip of mother-of-pearl passing through the centre of the field. The picture in the focus of the eye-glass is broken into two parts, and the view is rendered still more unpleasant by the inequality of the segments into which the field is divided. In addition to these disadvantages, the different divisions of the micrometer are at unequal distances from the eye-glass which views them, and therefore can neither appear equally distinct, nor subtend equal angles at the eye.

The circular mother-of-pearl micrometer, is free from all these disadvantages, and has likewise the benefit of a kind of diagonal scale, increasing in accuracy with the angle to be measured. This micrometer, which I have often used, both in determining small angles in the heavens, and such as are subtended by terrestrial objects, is represented in Plate III. Fig. 4, which exhibits its appearance in the focus of the first eye-glass. The black ring, which forms part of the figure, is the diaphragm, and the remaining part is an annular portion of mother-of-pearl, having its interior circumference divided into 360 equal parts. The mother-of-pearl ring, which appears connected with the diaphragm, is completely separate from it, and is fixed at the end of a brass tube, which is made to move between the third eye-glass and the diaphragm, so that the divided circumference may be placed exactly in the focus of the glass next the eye. When the micrometer is thus fitted into the telescope, the angle subtended by the whole field of view, or by the diameter of the innermost circle of the micrometer, must be determined either by measuring a base, or by the passage of an equatorial star; and the angles subtended by any number of divisions or degrees will be

found by a table constructed in the following manner.

Let A $mpnb$, Plate III. Fig. 5. be the interior circumference of the micrometer scale, and let mn be the object to be measured. Bisect the arch mn in p, and draw Cm, Cp, Cn. The line Cp will be at right angles to mn, and therefore mn will be twice the sine of half the arch mpn. Consequently, AB : mn = Rad : Sine of $\frac{1}{2}mpn$; therefore $mn \times \mathrm{R} = \sin. \frac{1}{2}mpn \times \mathrm{AB}$, and $mpn = \frac{\sin. \frac{1}{2}mpn}{\mathrm{R}} \times \mathrm{AB}$; a formula by which the angle subtended by the chord of any number of degrees may be easily found. The first part of the formula, viz. $\frac{\sin. \frac{1}{2}mpn}{\mathrm{R}}$ is constant, while AB varies with the size of the micrometer, and with the magnifying power which is applied. We have therefore computed the following Table, containing the value of the constant part of the formula for every degree or division of the scale.

Degrees.	Constant part of the Formula.	Degrees.	Constant part.	Degrees.	Constant part.	Degrees.	Constant part.	Degrees.	Constant part.
1	.0087	37	.3173	73	.5948	109	.8141	145	.9537
2	.0174	38	.3256	74	.6018	110	.8192	146	.9563
3	.0262	39	.3338	75	.6088	111	.8241	147	.9588
4	.0349	40	.3420	76	.6157	112	.8290	148	.9613
5	.0436	41	.3502	77	.6225	113	.8339	149	.9636
6	.0523	42	.3584	78	.6293	114	.8387	150	.9659
7	.0610	43	.3665	79	.6361	115	.8434	151	.9681
8	.0698	44	.3746	80	.6428	116	.8480	152	.9703
9	.0785	45	.3827	81	.6494	117	.8526	153	.9724
10	.0872	46	.3907	82	.6561	118	.8572	154	.9744
11	.0958	47	.3987	83	.6626	119	.8616	155	.9763
12	.1045	48	.4067	84	.6691	120	.8660	156	.9781
13	.1132	49	.4147	85	.6756	121	.8704	157	.9799
14	.1219	50	.4226	86	.6820	122	.8746	158	.9816
15	.1305	51	.4305	87	.6884	123	.8788	159	.9833
16	.1392	52	.4384	88	.6947	124	.8829	160	.9848
17	.1478	53	.4462	89	.7009	125	.8870	161	.9863
18	.1564	54	.4540	90	.7071	126	.8910	162	.9877
19	.1650	55	.4617	91	.7133	127	.8949	163	.9890
20	.1736	56	.4695	92	.7193	128	.8988	164	.9903
21	.1822	57	.4771	93	.7254	129	.9026	165	.9914
22	.1908	58	.4848	94	.7314	130	.9063	166	.9925
23	.1994	59	.4924	95	.7373	131	.9100	167	.9936
24	.2079	60	.5000	96	.7431	132	.9135	168	.9945
25	.2164	61	.5075	97	.7490	133	.9171	169	.9954
26	2250	62	.5150	98	.7547	134	.9205	170	.9962
27	.2334	63	.5225	99	.7604	135	.9239	171	.9969
28	.2419	64	.5299	100	.7660	136	.9272	172	.9976
29	.2504	65	.5373	101	.7716	137	.9304	173	.9981
30	.2588	66	.5446	102	.7771	138	.9336	174	.9986
31	.2672	67	.5519	103	.7826	139	.9367	175	.9990
32	.2756	68	.5592	104	.7880	140	.9397	176	.9994
33	.2840	69	.5664	105	.7934	141	.9426	177	.9996
34	.2923	70	.5735	106	.7986	142	.9455	178	.9998
35	.3007	71	.5807	107	.8039	143	.9483	179	1.0000
36	.3090	72	.5878	108	.8090	144	.9511	180	1.0000

In order to find the angle subtended by any number of degrees, we have only to multiply the constant part of the formula corresponding to that number in the table by AB, or the angle subtended by the whole field. Thus, if AB is 30 minutes, as it happens to be in the micrometer which I have constructed, the angle subtended by one degree of the scale will be $30' \times .0087 = 15\frac{3}{7}$ seconds, and the angle subtended by 40 degrees will be $30' \times .342 = 10'\ 15''.6$; and by making the calculation it will be found, that as the angle to be measured increases, the accuracy of the scale also increases; for when the arch is only 1 or 2 degrees, a variation of 1 degree produces a variation of about 16 seconds in the angle; whereas when the arch is between 170 and 180, the variation of a degree does not produce a change of much more than one second in the angle. This is a most important advantage in the circular scale, as in Cavallo's micrometer a limit is necessarily put to the size of the divisions.

It is obvious, from an inspection of Fig. 4. Plate III. that there is no occasion for turning the circular micrometer round its axis, because the divided circumference lies in every possible direction. In Fig. 5, for example, if the object has the direction *ab*, it

will be measured by the arch *aob*; and if it lies in the line *cd*, it will be measured by the arch *crd*.

In the circular micrometer which has been mentioned, AB, or the diameter of the field of view, is exactly half an inch, the diameter of the brass tube in which it is fixed one inch, the length of the tube half an inch, and the degrees of the divided circumference $\frac{1}{230}$th of an inch.

CHAP. VI.

Description of a Rotatory Micrometer with Points.

In this instrument, a fine steel point s, Plate III. Fig. 5, is firmly fixed in the anterior focus of the first eye-glass of a telescope, so as to be distinctly visible in the field of view. Another steel point t of the same kind projecting into the field, is fixed to that part of the eye-tube which contains the first glass. This tube has a motion of rotation about the axis of the eye-piece, by means of which the moveable point t may be brought to coincide with the fixed point s; and may be separated from it in the circumference of a circle. The separation, or the distance of the two points, is always equal to the chord st of the arch $s\mathrm{B}t$ which lies between them; and consequently the angle subtended by the two points in one position, is to the angle which they subtend in any other position, as the chords of the arches through which the move-

able point must travel to make the two points coincide. In order to find these arches, therefore, the eye-piece of the telescope carries a circular head CD, Plate III. Fig. 6, divided into 360°, and firmly fixed to the eye-tube T. From the centre of the head there rises an annular shoulder, concentric with the tube, and containing a diaphragm with the fixed point. The part of the eye-piece AB which carries the first eye-glass, contains also a diaphragm with a steel point, and is screwed into the shoulder O of the part of the circular head EF, which carries the vernier V. The ring EF is fitted in such a manner as to move upon the annular shoulder rising from CD;—so that by turning round the tube AB, a rotatory motion is communicated to the vernier V, and to the steel point in one of the diaphragms, while the other steel point remains immoveable.

In using this instrument, therefore, we have only to direct the telescope to any object, and separate the steel points till they exactly cover the two points which contain the angular distance required; and as the index will point to 0 degrees when the points coincide, it will now shew upon the graduated head the arch which lies between the points. The angle corresponding to this arch may be found by means of the Table in the pre-

ceding Chapter, in the manner which has been already explained.

When the angle corresponding to the diameter of the circle in which the moveable point revolves has been experimentally determined, the angles themselves, expressed in minutes and seconds, might be inserted on the graduated head AB.

It is obvious, that the moveable point *t* may be separated from the fixed one, by many other contrivances than the mere circular motion of the eye-tube. The method represented in the figure is the most simple, and perhaps the best, when the centering is well executed; but if we wish to enlarge the scale, the separation of the points may be effected, by means of a pinion working in the teeth of a wheel, to which the moveable point is attached. The fixed point might perhaps be advantageously placed in the focus of the whole eye-piece, while the moveable one revolves in the focus of the first eye-glass, so that both the points may be seen with perfect distinctness,—an effect which cannot be obtained with complete accuracy, when they are both in the anterior focus of the same lens, and when, at the moment of coincidence, the one is behind the other.

The same principle which is employed in the construction of this instrument, may be applied

to a divided object-glass, the one semilens having a motion of rotation round the other; so that the distance between their centres, and consequently the angle subtended by the line joining the centres of the images, may constantly vary during a complete revolution.

CHAP. VII.

*Description of an Eye-piece Wire Micrometer.**

The general principle upon which this instrument is founded, is the same as that which we have explained in the first Chapter of this Book. In the micrometer which is there described, the angle of a pair of fixed wires is increased and diminished, by the motion of a second object-glass along the axis of the telescope; while, in the pre-

* An instrument of nearly the same kind with that which is the subject of the following Chapter, has been described by Mr Ezekiel Walker, in the Philosophical Magazine for August 1811, Vol. xxxviii. p. 127, as a new invention of his own. I certainly cannot suppose, that this ingenious writer had an opportunity of seeing any of the instruments of this kind, which had been constructed under my direction. So early as the end of the year 1805, I sent a drawing and description of the eye-piece micrometer to Mr Carey, optical instrument maker, London. In 1806, one of the instruments made for me by Messrs Miller and Adie, Edinburgh, was examined by Professor Playfair; and since that time it has been in the possession of a friend in London. The only variation in the instrument proposed by Mr Walker is, the use of the lines on a slip of mother-of-pearl, instead of the silver wires.

sent instrument, the variation of the angle is effected by separating the two parts which compose an achromatic eye-piece; or when the eye-piece consists of two or three lenses, by separating the lens next the eye from the remaining lenses. If the small tube, which contains the field-glass and the first eye-glass, be pulled out beyond its natural position, the magnifying power of the instrument will be increased; and if the same tube be pushed farther in than its natural position, the magnifying power will be diminished. It will be found in general, that if the tube already mentioned be allowed to move over the space of four inches, that is, about two inches on each side of its natural position, the magnifying power at one extremity of this space will not be very far from double of what it is at the other extremity.

The eye-piece micrometer is represented in Plate III. Fig. 7, with all the lenses in their natural position. The part AFG, containing the two lenses A, C, is fixed to the telescope, and a space is left between the tube AC and the outer tube, in order to permit the moveable part of the eye-piece to get sufficiently near to the lens C, and also to a sufficient distance from it. The other tube DB, containing the field-glass D, and the first eye-glass B, is moved out and in by a rack and pinion E. The scale

is engraven upon the upper surface $l\,n$, and the divisions are pointed out by the index of a vernier placed at the extremity m of the outer tube FG. The zero of the scale is the point marked out by the index of the vernier, when the tube DB is pushed in as far as possible; and the divisions may be read off, if necessary, by means of a convex glass at F, fixed to the tube AFG.

The nature of the scale, by which the variations of the angle are measured, might be accurately determined by direct experiment; but as it may be more convenient to fix the value of the divisions by calculation, we shall now point out the method by which this may be done.

Let us first suppose, that the eye-piece consists of two lenses, placed at a distance greater than the sum of their focal lengths, as in Plate III. Fig. 8, where O is the image formed by the telescope, B the first, and A the second eye-glass. Let the image formed by the lens A be at o, B o will be the focal length of B, when the image is seen distinctly through the telescope. By pulling out the lens B to B′, the image o must be formed at o', when it is seen distinctly in this new position, B′ o' being equal to B o; and the conjugate focal length of A will now be A o', instead of A o. But, in order that an image of O may be formed at o', the whole eye-piece, must be brought

nearer to O, or O must be brought nearer to A, so that their distance is AO′. The change of magnifying power, produced by the lens A alone, is, in the first position of the lenses, equal to $\frac{A\,o}{AO}$, or (calling $A\,o = a$, and $AO = d$) $= \frac{a}{d}$; and, in the second position, $= \frac{A\,o'}{AO'} = \frac{a'}{d'}$. Now, if $f =$ focal length of A, we have, by the principles of optics, $d = \frac{a\,f}{a-f}$, and dividing a by this expression, we have $\frac{a^2 - a\,f}{a\,f} = \frac{a-f}{f} = \frac{a}{f} - 1$ for the magnifying power, as produced by the lens A alone. Now, calling $f = 2$, $a = 6$, $a' = 7$, $a'' = 8$, we shall have the magnifying power $= 2$, $2\frac{1}{2}$, 3, and the angles are inversely proportional to the magnifying powers. Calling the greatest angle 10, we shall have the results in the following Table:

Value of a, or the conjugate focal length of A.	Calculated magnifying powers.	Differences of magnifying power.	Calculated angles.	Differences of angles.	Angles if the scale were one of equal parts.
6	2.0		10		10
		0.5		2	
7	2.5		8		8.75
		0.5		1.333	
8	3.0		6.666		7.50
		0.5		0.952	
9	3.5		5.714		6.25
		0.5		0.714	
10	4.0		5		5

From these results, it appears, that the scale which measures the variation of the angle is not a scale of equal parts; that is, equal changes in a, or in the position of the lens B, do not correspond to equal variations of the angle.

If there are three lenses in the eyepiece, as in Plate III. Fig. 9., and if the variation of power is produced by the motion of the third lens C, the conjugate focal length of the lens B will obviously vary with the motion of C, as the image must always be formed at o, the anterior focus of C. Let us now call $B\,o = a$, $f =$ focal length of B, $m =$ BA, F = focal length of A, $d =$ BS, the distance from B of the point S, from which rays must diverge, in order to be refracted to o, or, which is the same thing, the distance from B of the point S, or virtual focus of the diverging rays after they are refracted by the lens A; and D = the distance of the image O, formed by the object glass from the first lens A. Since the rays proceeding from O diverge after refraction through A, as if they proceeded from S, and are afterwards converged to o by the lens B, it is manifest that o and S are conjugate foci of the lens B; so that we have, by the principles of optics, $BS = d = \frac{af}{a-f}$. Then, since $AS = d - m$, and since S is the virtual focus for rays diverging from O,

a point nearer to A than its principal focus, we have

$$AO = D = \frac{AS \times F}{AS + F}, \text{ or, AS being} = d - m,$$

$$D = \frac{F \times \overline{d - m}}{d - m + F},$$

but the magnitude of the image at o, is, to the magnitude of the image at O, as $a : D$; hence, the magnifying power produced by the two lenses A, B, will be $= \frac{a}{D}$, or the magnifying power

$$= \frac{a \times \overline{d - m + F}}{F \times \overline{d - m}}.$$

Now, supposing $F = 15$, $f = 20$, $a = 30, 35, 40$, &c. $m = 23$, and the greatest angle $= 10$, we shall have the results in the following Table.

Different values of a.	Values of d.	Calculated magnifying powers.	Differences of mag. power.	Calculated angles.	Differences of angles.	Angles if the scale were of equal parts
30	60	2.81	1.00	10	2.63	10
35	46.666	3.81	1.21	7.37	1.77	8.36
40	40	5.02	1.44	5.60	1.25	6.72
45	36	6.46	1.71	4.35	.91	5.08
50	33.333	8.17		3.44		3.44

It appears from these results, that the scale which measures the changes of the angle is not a scale of equal parts.

When another lens is interposed between B and its conjugate focus o, as in Fig. 9., so as to constitute the common achromatic eyepiece, the magnifying power produced by these three lenses will be $= \frac{\varphi}{x + \varphi} \times \frac{a \times \overline{d - m + F}}{F \times \overline{d - m}}$, where φ is the focal length of the 3d lens, and x its distance from the fieldbar. In this case, the scale will be the same as in the last case, $\frac{\varphi}{x + \varphi}$ being a constant quantity, since the distance between the field-glass and eye-glass is invariable.

We have already said, that the value of the scale of this micrometer might be advantageously determined by direct experiment. This may be done very accurately by the method which has already been described in Chap. I. of this Book, by observing how much of an object at a given distance is comprehended between a pair of wires, when the moveable part of the tube is pulled out 1, 2, 3, 4, &c. divisions of the scale successively. The intervals thus embraced by the wires will give the proportional value of each unit of the scale; so that, by simple proportion, their real value in minutes and seconds may be found for different pairs of wires.

The following method, however, is more simple,

and perhaps equally accurate. After having found the greatest angle subtended by a pair of wires, placed in the focus of the eyeglass, or the angle when the index is at the zero of the scale, by the method in Chap. I., take the eye-piece out of the telescope, and having pushed the tube which contains the moveable lens or lenses as far in as possible, direct it as a microscope to a scale minutely divided.* Mark the position of the index when the wires comprehend exactly a certain number of these divisions, say 50, which they may be made to do, by a very trifling motion of the moveable tube, and make this point the zero of the scale. Let the moveable tube be now pulled out till the wires successively comprehend 48, 46, 44, 42, &c. of the divisions, or any other numbers, diminishing in arithmetical progression, and mark these points upon the tube. By this means, a scale will be formed, in which the divisions correspond to equal variations in the angle. If it should be found convenient to divide the scale into equal parts, the value of the divisions may be found in the same way.

The principle of the eye-piece micrometer may

* The beautiful micrometrical scales constructed by Mr Coventry, are admirably adapted for this purpose.

be applied with some advantage in the construction of a double image micrometer. The two images may be formed either by two semilenses fixed at an invariable distance, and substituted in the place of the principal object glass, or by bisecting the third eye-glass, and fixing its segments at a given distance. The angle subtended by the centres of these double images, would then be varied by the motion of part of the compound eye-glass, and this variation would be measured by a scale constructed in the way which has already been described.

In the seven preceding Chapters, we have briefly described several new micrometers, which appear to possess considerable advantages. We shall now notice some other principles of construction, which may on some occasions be successfully employed in the mensuration of small angles, though they are not perhaps of sufficient importance to demand a separate Chapter.

A very simple micrometer, with double images, may be constructed by dividing the third eye-glass of an achromatic eye-piece, and fixing the seg-

ments at an invariable distance. The angle formed by the line joining the centres of the two images, may then be varied at pleasure, either by separating one half of the eye-piece from the other half, or, what is still better, by moving a lens between the object-glass and its principal focus. If the first method is adopted, the angle will vary in the same manner as in the wire micrometer described in Chapter I.; and if the second method is employed, the variation of the angle will be the same as in the eyepiece micrometer described in this Chapter.

Another micrometer, with double images, may also be formed, by bisecting the small mirror of a Gregorian or Cassegrainian telescope, and making the angle subtended by the centres of the two images vary by the motion of the eye-piece, as in the micrometer described in Chapter II.

In microscopical observations, where it is required to measure the magnitude of minute objects, the angle which they subtend may be varied by immersing the objects themselves, as well as the object-glass of the microscope, in some fluid whose refractive power can be easily changed by the addition of another fluid. In consequence of this variation of refractive power, the magnifying power

of the microscope suffers a corresponding change; and therefore a pair of wires, or any other body placed either in the focus of the eye-glass, or in the focus of the microscope, will subtend different angles at the eye of the observer. The proportion of the two fluids becomes, in this case, the scale which measures the angular variations. This principle, which has a very extensive application in the construction of instruments, will be more fully explained in Book V.

CHAP. VIII.

On the Application of some of the preceding Micrometers to Microscopes.

The attempts which were first made to measure the size of microscopical objects, were extremely rude and imperfect. Leeuwenhoek estimated the magnitude of minute bodies by comparing them with small grains of sand, one hundred of which were equal to an inch in length. Dr Jurin made a similar use of small pieces of silver wire, the diameter of which he determined by wrapping the wire round a cylindrical pin, and observing how many breadths of it were equal to an inch. The most valuable contrivances, however, of this nature, are the beautifully divided scales which Mr Coventry has formed upon ivory, glass, and silver. These scales are constructed with a degree of minuteness and accuracy which have never been surpassed, and are of the greatest use in

microscopical observations, and in many physical researches.

The first application of the screw to a microscope, was made by Mr Benjamin Martin. A long and slender screw was made to traverse the field of the microscope at the anterior focus of the eye-glass; and the size of any part of an object over which its extremity passed, was measured by the number of revolutions and parts of revolutions which the screw had performed,—the part of an inch corresponding to any number of revolutions having been previously ascertained by experiment.

The application of the screw has been brought to great perfection in the micrometer microscopes, which have of late years been employed for reading off the divisions on the limbs of circular instruments, and for measuring the dilatation and contraction of bodies by heat and cold; but almost no attention has been paid to the construction of simple and accurate micrometers for measuring the size of microscopical objects.

The principle employed in the new wire micrometer is obviously applicable to the compound microscope. The variation of magnifying power may be effected either by giving the amplifying lens a motion along the axis of the microscope,

while the object-glass and eye-glass are stationary,—or this motion may be communicated to the eye-glass and amplifying-glass, placed at a constant distance in the same tube. A scale of known dimensions, engraven upon glass, should then be placed in the anterior focus of the eye-glass, so that the divisions may be seen distinctly when the instrument is adjusted to the microscopic object. If the part of the object which we wish to measure does not occupy any exact number of the units upon the scale, the angle which it subtends may be increased or diminished by altering the position of the moveable lens or lenses, till the part of the object exactly coincides with a certain number of units. The space described by the lenses will then be a measure of the variation of the angle, and consequently of the size of the object; the magnitude of one or more of the divisions on the scale having been previously ascertained by experiment. As some light must be lost by the interposition of the glass scale, the circular mother-of-pearl micrometer might be placed in the anterior focus of the eye-glass, and used in the manner already described in Chapter V. of this Book; and each unit of the scale may be again

subdivided by the method which we have now been explaining.

Of all the micrometers, however, which can be applied to the microscope, the rotatory micrometer (see Chapter VI.) is by far the most simple. It is constructed in the same way for a microscope as for a telescope; and when we have once determined experimentally the space which corresponds to the greatest distance of the two steel points, the space which they comprehend, at any other position, may be easily found. In using this instrument, therefore, we have only to separate the steel points till they embrace the object, or part of an object, which it is required to measure; and, by the angular distance of the points in degrees and minutes, as observed upon the circular divided head, we may find from the table in p. 52, the length of the corresponding chord, and consequently the magnitude of the object required.

The expansion of luminous points into circular images, might also be employed to measure the magnitudes of minute objects; and the principle of double images, as explained in Chapter II., might be applied with equal success. *

* See the last paragraph of the preceding Chapter.

CHAP. IX.

On the Fibres proper for Micrometers, and on the Method of Adjusting them in the Focus of the Eye-glass.

In the micrometer constructed by Huygens, the object whose angle was required, was comprehended between the edges of two plates of brass. Silver wires, and sometimes hairs, were afterwards substituted instead of the plates, and continued in use till about the end of the last century. The finest silver wire ever made was drawn in France to the thickness of $\frac{1}{1000}$ of an inch. The plates used for this purpose, and the secret of making them, are said to have been lost amid the convulsions of the revolution. The smallest silver wire which is made in this country does not exceed $\frac{1}{650}$ of an inch.

The impossibility of obtaining wire of a diameter sufficiently minute for micrometers, induced

Felix Fontana,* in 1775, to recommend the spider's web as an excellent substitute for silver wire. According to La Lande, Fontana found some of these fibres so small as the 8000th part of a line, † or the 96,000th part of an inch, and he always employed those that were newly made. This suggestion of Fontana, however, did not excite much notice, till the use of the spider's web was introduced by the celebrated Mr Edward Troughton, who found this fibre to be so fine, opaque, and elastic, as to answer all the objects of practical astronomy. We are informed, however, by this distinguished artist, that it is only the stretcher, or the long line which supports the web, that possesses these valuable properties. The other parts of the web, though equally fine and elastic, are very transparent, and therefore completely unfit for micrometrical fibres. The difficulty of procuring the proper part of the spider's web, has compelled many opticians and practical astronomers to employ the raw fibre of unwrought silk, or, what is much worse, the coarse silver wire which is manu-

* *Saggio del real gabinetto di fisica e di storia naturale de Firenze*, Rom. 1775, 4to, quoted by La Lande.

† We suspect that some error has been committed in this measurement. The 96,000th part of an inch is too minute a portion of space to be seen distinctly even by very powerful microscopes.

factured in this country. But whatever be the comparative advantages of these different substances, they are all liable to the error arising from the inflexion of light, which renders it impossible to ascertain the exact contact between the fibre and the luminous body. This disadvantage has been experienced by every astronomical observer, and has always been considered as inseparable from the wire micrometer. After numerous trials, I have succeeded in obtaining a delicate fibre, which appears to remove the error of inflexion, while it possesses the requisite properties of opacity and elasticity. This fibre is glass, which is so exceedingly ductile that it can be drawn to any degree of fineness, and can always be procured and prepared with facility,—a circumstance of no small importance to the practical astronomer, who is frequently obliged to send his micrometers to a great distance when they require to be repaired.

When the vitreous fibre is formed, and stretched across the diaphragm of the eye-piece of a telescope, it will appear perfectly opaque, with a delicate line of light extending along its axis. This central transparency arises from the refraction of the light which falls upon the edges of the cylindrical fibre, and therefore the diameter of the luminous streak must vary with that of

the fibre itself. In a micrometer which we have fitted up in this way, the glass fibres are about the 1200th part of an inch in diameter, and the fringe of light which stretches across their axis, is distinctly visible, though it does not exceed in diameter the 3000th part of an inch. In using these fibres for measuring the angle subtended by two luminous points, whether they be two stars, or the opposite extremities of a luminous disc, we may, as has hitherto been done, separate the fibres till the luminous points are in contact with their interior surfaces; but, in order to avoid the error arising from inflexion, we would propose that the separation should be continued till the rays of light issuing from the luminous points dart through the transparent axes of the fibres. The rays, thus transmitted, suffer no inflexion in passing through the fibre to the eye; and besides this advantage, we have the benefit of a delicate line, about one-third of the diameter of the fibre itself.

On some occasions we have employed threads of melted sealing-wax, which may be made extremely fine, though not of such a regular diameter as silver wire, or the fibres of glass; and it is a very singular fact, that one of these fibres of wax was exposed, without injury, to the heat of the sun, concentrated in the focus of an object-

glass, with an aperture of 2.3 inches, and 29 inches in focal length.

These fibres are placed in delicate parallel grooves formed upon the diaphragm of the first eye-glass, and may be fixed in their places, for temporary purposes, by a thin layer of bees-wax;—but when they are required to be kept at an invariable distance, it is safer to pinch them to the diaphragm by a small screw-nail near the extremity of each wire.

The diaphragm should be constructed so as to move along the axis of the eye-piece, in order that the fibres may be placed exactly in the anterior focus of the first eye-glass; and before any observation is made, the eye of the observer ought to be fixed for a short time upon the fibres alone, till it is accommodated to that distance, and while it is thus fixed, distinct vision should be produced by the motion of the eye-tube. By attending to this suggestion, which is of great practical importance, the rays that diverge from the fibres, and those that diverge from the distinct image, will unite on the same points of the retina. From the great facility which the eye possesses of adjusting itself to different distances, the adaptation of that organ to the fibres and to the image, could not have been effected by looking at the image alone, while dis-

tinct vision was produced by the motion of the eye-piece, for as the eye has not any permanent focus like a lens, the image might appear distinct before the pencils of rays had actually converged to a point; and when this does happen, the rays proceeding from the fibres cannot unite with those proceeding from the image, on the same points of the retina.

When the micrometer is employed in terrestrial observations, the end of the eye-tube into which the observer looks, should be furnished with an aperture smaller than that which is used for common purposes, and this small aperture should be used when the sun shines, or when the light of the day is very great. If the fibres happen to be small, they will either cease to become visible in very strong light, or will appear to have a kind of vibratory motion which injures the eye of the observer, and prevents him from making the observation. Hence it becomes necessary to diminish the light by means of a small aperture.

BOOK II.

ON
INSTRUMENTS FOR MEASURING ANGLES
WHEN THE
EYE IS NOT AT THEIR VERTEX.

FROM the subject of micrometers for measuring the angle subtended at the eye of the observer by a line joining any two points, we are naturally led to consider the construction of instruments for measuring angles, when the eye is not placed at their vertex. Instruments of this kind have an extensive and important application in the arts and sciences; and while they may be employed for a variety of inferior purposes, they are particularly useful for measuring the angles of crystals, the inclination of strata, the declivity of mountains, and the horary angles of the fixed stars.

The principle upon which several of these instruments are formed, is derived from the simple

theorem in optics, that the images of objects, formed by reflection from a plain surface, have the same position, with respect to the reflecting surface, as the objects themselves. If the object, therefore, be a straight line, the angle which it forms with the reflecting plane will be equal to the angle which the image forms with the same plane, and the angle contained by the straight line and its image will be double of either of these angles. Hence it follows, that when the angle which the straight line forms with its image is equal to two right angles, or when the object and its image form one line, the straight line will be at right angles to the reflecting surface. This is obvious from Plate V. Fig. 1., where CD is the reflecting plane, and AB a straight line inclined to it at any angle. The angle DB *a*, which its image B *a* forms with the plane CD, will in every position be equal to the angle DBA, which the object forms with the same plane; and therefore, when the object A'B, and its image B *a'*, form one straight line A' *a'*, the line A'B is perpendicular to the reflecting surface; for the angle A'BD being equal to *a'* BD, they are both right angles. If the straight line AB forms the acute angle ABD with the reflecting surface, and is viewed by the eye in the direction CD, it will then ap-

pear to form one line with its image A a, and will be at right angles to the line $c\ d$, which is perpendicular to CD. Hence we may conclude, in general, that when any straight line appears to form one line with its reflected image, it is at right angles to a line in the reflecting surface, perpendicular to the axis of the eye. If we now suppose the reflecting surface to receive a rotatory motion in a plane passing through $c\ d$ at right angles to the axis of the eye CD, till any rectilineal object forms the same straight line with its image, we then obtain the relative position of the object to the reflecting plane; and the inclination of this plane being measured upon a graduated arch, gives us the relative position of the object to the horizon. By determining, in this manner, the respective inclinations of two lines which cut each other, to the horizon, or to any other plane, and by taking the difference of these inclinations, we obtain the angle which they form with one another.

The application of this principle constitutes the Klinometer for measuring, with the naked eye, the apparent inclination of strata, &c. ;—the goniometrical microscope for determining the angles formed by lines, or by the common sections of the planes of crystals ; and the goniometrical te-

lescope, for measuring the apparent inclination of lines when great accuracy is required, or when the object is too distant to be examined by the naked eye.

If, instead of measuring the apparent inclination of a line to the horizon, we wish to ascertain the angle formed by two reflecting surfaces, we have only to give a motion of rotation to the reflecting surfaces in a plane perpendicular to their common section, and mark, by means of the continuity of the object and its image, the positions in which any distant line is at right angles to a line in the reflecting surface, perpendicular to the axis of the eye. The arch intercepted between these two positions, is the supplement of the angle formed by the planes. The same result would have been obtained, if the reflecting surfaces had been stationary, and a rotatory motion communicated to the rectilineal object. If AB and BC, Plate V. Fig 2., for example, are sections of the reflecting surfaces, and MN, OP, the right lined object in two positions, and perpendicular to the surfaces AB, BC, then the angle MDO, or the arch MO, is the supplement of the angle ABC; for, in the quadrilateral figure NBPD, the angles at N and P are right, and, therefore, the angles at B and D are, together, equal to two right an-

gles; consequently the angle MDO is the supplement of the angle ABC. Upon this principle depends the goniometer for measuring the angles of crystals, which will be fully described in a subsequent chapter.

CHAP. I.

Description of a Klinometer.

THIS instrument, which is intended to measure with the naked eye the apparent inclination of strata,* the declivity of mountains, and the apparent magnitude of angles, when the eye is not at their vertex, is represented in Plate IV. Fig 1., where AB is a graduated quadrant, about three inches radius, fixed on the base BF, ED. The arm CM, which carries the vernier scale MN, moves round the centre C of the quadrant. This arm likewise carries a frame GH, inclosing a reflecting plane, made of black glass, or any other reflecting substance, which moves upon pivots *a* and *b*, in a plane perpendicular to that of the quadrant. When the instrument is to be used, it is adjusted to a horizontal plane by the

* A very ingenious Klinometer for measuring the dip and the bearing of strata, when the observer is able to reach them, has been invented by Lord Webb Seymour.

level LL and the three finger screws D, E, F. The moveable arm is then shifted along the graduated arch, till the reflected image of the stratum seen in the black mirror is continuous with the direct image. The arch, intercepted between the index of the vernier and the beginning of the scale, will shew the angle of inclination. The reason of this is obvious from Plate V. Fig. 3., where CD is the stratum, and EF the reflecting plane. The triangles DBE and DCE are obviously similar, and hence the inclination of the stratum, or the angle DBE is equal to the angle DEC, and is consequently measured by the arch AF. If we wish to determine the angle formed by two lines, there is no occasion for adjusting the instrument to a horizontal plane, as the value of the angle is equal to the difference of the respective inclinations of the lines to any given plane.

It is frequently of advantage, in using this instrument, to have the frame GH and the piece of black glass made as thin as possible, that when the eye is placed near the frame GH, and is looking obliquely into the reflecting surface, it may perceive a greater portion of the direct image of the line DB, whose inclination is required. When this is the case, the reflected image of DB, seen by the upper half of the pupil, will appear to co-

incide with the direct image, seen by the lower half of the pupil; and, by this means, the observer will be enabled to determine more accurately, when the line DB is perpendicular to the reflecting plane, than he could have done by judging merely of the apparent continuity of that line and its reflected image.

It is obvious that the reflecting plane GH may be placed on any part of the radius CM, on the centre above C, or even on the circumference of the graduated arch, without altering the principle of the instrument.

The preceding instrument might be of considerable advantage in laying down, in accurate perspective, a building, or any other object, in which there are a number of straight lines. The relative position of all these lines, either to the horizon or to any other line, could be be quickly ascertained.

CHAP II.

Description of a new Goniometer for measuring the Angles of Crystals.

THE Goniometers which were formerly employed to measure the angles formed by the planes of crystals, consisted of two small levers, having one arm considerably shorter than the other, and moving round the same centre of motion. The two short arms of the lever were opened, and placed upon the planes of the crystal, in a line perpendicular to the common section of the planes. The angle formed by the levers being equal to the angle of the planes, the centre of motion was placed upon the centre of a graduated arch, whose radius was equal to the longest of the levers, and the extremities of these arms pointed out the angle required upon the divided circumference. The inaccuracy of measurements made in such a rude manner was so great, that the angles formed by the planes of crystals, as given by different writers on crystallography, often differed by one, two,

and sometimes three degrees; and hence it became a matter of great importance to construct a more perfect instrument, by which these angles could be measured to any degree of accuracy. The necessary imperfection of every mechanical method, induced me to try the principle of reflection which has already been explained; and in applying it to a goniometer for measuring the angles of crystals, I have succeeded beyond my most sanguine expectations.* The methods of employing this instrument, which we shall now describe,

* Though the goniometer for measuring the angles of crystals is sufficiently different from the ingenious reflective goniometer recently invented by Dr Wollaston; yet, to those who may think that there is any resemblance between the two instruments, it will be necessary to make the following statement: The principle upon which all the reflecting instruments, described in Book II. are constructed, occurred to me more than five years ago, and appeared in a work which was published in September 1807. The fertility of this principle was obvious, and I was frequently employed in applying it to the construction of different philosophical instruments.

On the 3d of February 1809, I gave directions to Mr Harris to construct for me a goniometer, for measuring by reflection the angle which one line forms with another, or the angle formed by two reflecting surfaces, by observing their relative position to any straight line. I shewed this goniometer to several of my friends in London, during the months of February and March; to Dr Clarke and Mr Woodhouse at Cambridge, on the 22d of March; and, in the beginning of April, it was exhibited to the mathematical class of this university by Pro-

are applicable, not only to crystals whose surfaces are smooth and polished, but to crystals which reflect no light whatever, and even to those in such an imperfect state, that only small portions of the real surface occasionally appear.

The goniometer for measuring the angles of crystals is represented in Plate IV. Fig. 2. where a circle, AB, about six inches diameter, and divided into 360 degrees, moves round OO as a centre, and is supported by two upright bars M, N, fixed with screws into the stand SS. To the ring OO, supported by these bars, is fixed the arm G, that carries the vernier scale E. This scale remains stationary, while a rotatory mo-

fessor Leslie. At this time I got an addition made to the instrument by Mr Adie, by which it was fitted for measuring angles, when the eye was in their plane, by observing when the reflected image of an object came in contact with another object seen directly. The principal difference between this intrument and Dr Wollaston's Goniometer is, that in the latter the reflecting planes are placed at the centre of the graduated scale, whereas in the former they were placed on its circumference. In stating these particulars, I cannot be understood to mean that this philosopher, to whom the sciences owe so many obligations, was in any respect acquainted with the instruments which I had constructed. I am anxious only to shew, that the principle of all the instruments, described in this book, has been known to me for some years; and that most of the instruments themselves were actually constructed before Dr Wollaston's invention was made public.

tion is communicated to the divided circle AB by means of a pinion moved by the milled-head Q, which works in the teeth cut upon the circumference of the circle AB. A rectangular piece of brass L is fixed by two screws to one of the radii R of the graduated circle, so that the slider *s s* may move upon it, and be placed at different distances from the centre of motion, by laying hold of the pin below *s*. A thin plate *b c*, forming part of the cock *t b c* C on the top of this slider, carries the crystal, and by means of the handle *t* this plate has a motion round the centre C, in a plane perpendicular to that of the divided circle. Below this thin plate, and fixed to it by the screw C, is another piece of brass fastened to the top of the slider by the screw above C, and moveable, by means of the lever *l*, round that screw as a centre, in the same plane with the circle. When the handle *t* is employed to move the plate *b c*, it is pushed to or from the plane of the circle AB; but when the lever *l* is used to give the whole cock *b c* C a rotatory motion about the screw C, it is moved in a plane parallel to that of the circle AB. By the combination of these motions, the common section of the surfaces of the crystals is brought into a position parallel to the axis of the instrument. This adjustment

is effected by placing the graduated circle in such a position, that a vertical window-bar, or any other straight line, is nearly in the plane of the circle. A motion of rotation is then given to the crystal by the lever l;* and, if the reflected image of the window-bar forms one straight line with the object itself, when examined in each surface of the crystal, the adjustment is complete, or the plane of the graduated circle is parallel to a plane at right angles to the common section of the surfaces of the crystal. The instrument is then placed in such a position, that the plane passing through the eye and the window-bar is perpendicular to the plane of the divided circle, or that the common section of the surfaces of the crystal points to the bar of the window. The index is set to the beginning of the scale by means of a stop at the 180th degree, and the image of the vertical window-bar, or any rectilineal object formed by reflection from the first or right-hand surface of the crystal, is brought to coincide with the direct image by the vertical motion of the cock. The whole graduated circle is then made to revolve by the

* The rotatory motion of the crystal might be produced by means of a screw or pinion. By this means the adjustments would be more easily and accurately effected.

toothed pinion, till the reflected image of the vertical bar again coincides with the direct image when examined in the other surface of the crystal. When this position is obtained, the index of the vernier will point out, on the divided arch, the angle of the crystal.

If, during this operation, the vertical bar was exactly perpendicular to the planes of the crystal, the angle thus measured is the real angle formed by these planes, whatever was the distance of the crystal from the centre of motion; for the bar was perpendicular to every line lying in the two planes. When the bar, however, is inclined to the planes of the crystal, as AB, Plate V. Fig. 1. and appears to be perpendicular to them only when seen in one direction CD, then it is obvious, that there is a parallax depending on the angle ABA′, and on the horizontal aberration of the crystal from the centre of the graduated circle. This parallax, however, vanishes, when the common section of the planes of the crystal coincides with the axis of motion; and therefore it may be completely removed in the original construction of the instrument, by opening the centre of the divided circle, as in the figure. But as the parallax is so small, and may be removed by a simple adjustment, the instru-

ment would still measure the angle with great accuracy even if it were not open at the centre.

During the preceding operation, the goniometer must evidently remain steady on its base, as the smallest change in its position will affect the accuracy of the result. In order to remove this inconvenience, and render the instrument portable, and independent of any object exterior to itself, a horizontal arm DH, about six or eight inches long, projects from the base of the instrument, to which it is fastened by means of a finger screw and two projecting pins, and carries the vertical frame HK. Across this frame two or three pair of parallel silver wires are stretched in a perpendicular direction, and are used instead of the window-bars, in the way which has been already described. The angle of the crystal may then be determined when the instrument is held in the hand, without the accuracy of the measure being affected by any agitation whatever. In the mensuration of the angle, the middle pair of silver wires may be employed when the space described by the common section of the planes of the crystal is not great; but, when this space is considerable, one of the extreme pairs should be used with the first surface of the crystal, and the other pair with the second; for, if the silver wires

should not be exactly parallel to the plane of the graduated circle, the very small parallax arising from this cause will be nearly extinguished. The method of finding the coincidence between the wires seen directly, and their reflected image, will be understood from Plate V. Fig. 4. where *a b g*, *c d h* is a pair of silver wires, and *a b*, *c d* the portions of them that are above the surface of the crystal. When the eye is looking at their reflected image *b e*, *d f*, it can also, from its proximity to the crystal, see faintly the portions *b g*, *d h*, or at least their extreme portions at *g* and *h*. The crystal is therefore turned round by the handle *t*, if it is the first or right-hand surface; or by the nut opposite to Q, if it is the other surface that is used, till the reflected lines *b e*, *d f* exactly coincide with, or cover the wires *b g*, *d h*, or their extreme portions seen by direct vision.

If, instead of employing the frame HI, it is found convenient to use a distant object, we have only to make any point of that object seen by reflection from one of the surfaces of the crystal, come in contact with any point of another object seen by direct vision, and at a considerable distance from the first. The coincidence between the same objects is observed by means of the

other surface, and the angle of the crystal is determined as before.

It is obvious that this instrument will measure the angles of crystals with great accuracy, and little trouble, if the surfaces are moderately smooth, and reflect the smallest quantity of light. When the surface has the appearance of being perfectly rough and irregular, the oblique reflection generally gives a very distinct image of a vertical bar, when the image of a horizontal line or of any other object could not possibly be obtained. It frequently happens, however, that the crystal does not reflect sufficient light to form an image, or is so irregular in its surface, or so inconveniently placed in the specimen, that a variety of different contrivances must be adopted for measuring its angles. In a specimen of Allanite, for example, belonging to Mr Allan, the crystals are situated in such a manner, that their angles could not be measured, either by the goniometers of Hauy or Dr Wollaston, or by the instrument which has now been described, without breaking off some of the projecting parts of the mineral.

When the planes of the crystal are smooth, but unpolished, a small piece of parallel glass AB, Plate V. Fig. 5. or any other reflecting substance

with parallel sides, is successively placed upon the surfaces of the crystal CDE; the coincidence of the direct image of a rectilineal object with the image reflected from the piece of glass, is observed as before, and the angle found in precisely the same manner. If the two surfaces of the reflector should not be parallel, the aberration will be corrected by reversing its position on the second surface of the crystal.

When the planes of the crystal are covered with asperities which prevent the piece of glass from lying parallel to these planes, we must make use of the reflector AB, Plate V. Fig. 6. supported by three slender feet, and so formed that the reflecting plane *m n* is exactly parallel to the plane *o p*, passing through the extremities of the three feet. The three feet are then placed upon those points of the surface where there are no asperities, and the coincidence of the images is observed in the reflector: It is then transferred to the other surface of the crystal, the coincidence of the images again observed, and the angle of the planes measured as before. As the surface of the crystal may always be brought into a horizontal position when the coincidence of the object and its image is observed, the reflector will stand steadily on the planes of the crystal;

but, in order to secure it from sliding, a drop of varnish or melted bee's wax may be placed round each of its feet. It might be proper to have two or three of these reflecting tripods of different sizes, and with their feet at different distances, in order to accommodate themselves to the smooth parts of the crystal. One of the reflectors might be fixed on each surface with bee's wax, in the way represented in Plate V. Fig. 7. where C is the crystal, and A, B the two reflecting tripods. If the position of the crystal should prevent us from adopting either of these methods, which was the case in the specimen of Allanite already mentioned, we must have recourse to the goniometrical microscope, which is intended to measure the angles formed by two lines when the eye is perpendicular to the plane of the angle.

If we conceive the two surfaces of a crystal to be cut by a plane perpendicular to their common section, the apparent angle contained by the two lines which form the boundary of the section, when the eye is perpendicular to the section, is evidently the inclination of the planes. But if the cutting plane is not perpendicular to the common section, the apparent angle of the lines which form the boundaries of the section when viewed by an eye perpendicular to it, is evident-

ly greater or less than the real angle of the crystal, according to the position of the cutting-plane. If the observer, however, places himself in such a manner, that the common section of the planes is parallel to the axis of his eye, then the apparent angle formed by the bounding lines of the section, whatever be the position of the cutting-plane, is the real angle of the crystal. By placing the crystal therefore in this position, in the focus of the goniometrical microscope, which shall be hereafter described, and measuring the apparent angle formed by the bounding lines, we obtain, by a very simple process, the inclination of the planes.

This will be understood from Plate V. Fig. 8. in which ABCDEF is a crystal, ABC a section of it perpendicular to AD, and A*b c* an oblique section. Now, though BAC is the real angle of the crystal, yet, when the oblique section A*b c* is viewed by the observer at O, its bounding lines A*b*, A*c* are apparently coincident with the lines AB, AC, whose inclination is the real angle of the planes; and therefore, if we measure by a proper instrument, which we shall afterwards describe, the apparent angle contained by the oblique lines A*b*, A*c*, we obtain a measure of the real angle of the crystal.

The angles of the crystal may also be advantageously deduced, from the plane angles by which any of the solid angles is contained. The plane angles are first measured with great accuracy by the goniometrical microscope, or the angular micrometer adapted to a microscope, and the inclination of the planes is deduced from a trigonometrical formula. Whatever be the number of plane angles which contain the solid angle, we can always reduce the solid angle to one which is formed by three plane angles, and determine by the formula the inclination of any two of them. Thus, if the solid angle at A, Plate V. Fig. 9. is contained by five plane angles, and if it is required to find the inclination of the planes ABC, ACD, we first measure the plane angles CAB, CAD, and also the angle contained by the lines AB, AD; so that we have now reduced the solid angle contained by five plane angles, into one contained by three plane angles, CAB, CAD, BAD.

Legendre, in his Elements of Geometry, has given a very elegant solution of this problem by a plain construction; and it is easy from his solution to form an instrument for shewing the angles of the planes without the trouble of calculation. Thus let the angles BAC, CAD, DAE, Plate V. Fig. 10. be made equal to the three plane angles

by which the solid angle is contained. Make AB equal to AE, and from the points B, E let fall the perpendiculars BC, ED on the lines AC, AD, and let them meet at O. From the point C, as a centre with the radius CB, describe the semicircle BFG. From the point O draw OF at right angles to CO, and from F, where it meets the semicircle, draw FC. The angle GCF is the inclination of the two planes CAD, CAB. In order to construct an instrument on this principle, to save the trouble of projection or calculation, we have only to form a graduated circle BHEG, with three moveable radii, AC, AD, AE, and a fixed radius AB. The moveable radii must have vernier scales at their extremities, that they may be set so as to contain the three plane angles which form the solid angle. Two moveable arms BG, EO, the former of which is divided into any number of equal parts, turn round the extremities B, E; and, by means of a reflecting mirror on their exterior sides, they can be set in such a position as to be perpendicular to the radii AC, AD. When this is done, the number of equal parts between C and O, divided by the number between B and C, is the natural co-sine of the angle GCF; and therefore, by entering a table of

sines with this number, the inclination of the two planes will be obtained.

In order to obtain a more accurate result, however, we must have recourse to a trigonometrical formula. Let A, Plate V. Fig. 11. be the solid angle, and let it be required to determine, by means of the three plane angles, the inclination of the surfaces ACB, ACD. Draw AM, AN in the planes ACB, ACD, and perpendicular to the common section AC; join BM, DN. Then it it obvious, that the angle MAN is the inclination of the planes required, and that the angle BAD, which is an oblique section of the prism BM, will be equal to MAN when it is reduced to the plane AMN. By considering that the inclinations of the bounding lines of the oblique section of the prism, to the bounding lines of the perpendicular section, are measured by the angles DAN, BAM, the complements of the two given plane angles CAD, CAB, we shall obtain, by spherical trigonometry, the following formula:

$$\text{Sin.}\ \frac{\text{MAN}}{2} = \\ \text{R}\sqrt{\frac{\left(\text{Sin.}\ \frac{\text{BAD}+\text{CAD}-\text{CAB}}{2}.\ \ \text{Sin.}\ \frac{\text{BAD}+\text{CAB}-\text{CAD}}{2}\right)}{\text{Sin. CAB. Sin. CAD}}}$$

Or, calling φ the angle of the surfaces of the crystal, B, C the plane angles at the vertex of these

surfaces, and A the other plane angle, then we shall have

$$\text{Sin.}^2 \frac{\varphi}{2} = \text{Rad.}^2 \frac{\left(\text{Sin.} \frac{A+B-C}{2} . \ \text{Sin.} \frac{A+C-B}{2} \right)}{\text{Sin. B. Sin. C.}}$$

a formula from which the value of φ may be obtained by a very simple calculation.

Let the angle BAD, for example, be 62° 56′, the angle CAD = 100° 2′, and the angle CAB = 106° 10′, then we shall have, by the preceding formula,

$$\text{Sin.}^2 \frac{\varphi}{2} = \text{Rad.}^2 \frac{\text{Sin. } 28° \ 24' \ \text{Sin. } 34' \ 32''}{\text{Sin. } 106° \ 10' \ \text{Sin. } 100° \ 2'};$$

Now we have,

Log. sin. 28° 24′	- -	9.6772640
Log. sin. 34° 32′	- -	9.7534954
		19.4307594
Add 2 Log. of Rad.	-	20.0000000
		39.4307594
Log. sin. 106° 10′	- -	9.9824774
Log. sin. 100° 2′	- -	9.9933068
		19.9757842

From - - - - - -	39.4307594
Subtract - - - - -	19.9757842
2 Log. sin. $\frac{\varphi}{2}$ - - - -	19.4549752
Log. sin. $\frac{\varphi}{2}$ - - - - -	9.7274876

Hence $\frac{\varphi}{2} = 32° \; 16' \; 18''$

and $\varphi = 64° \; 32' \; 36''$

the angle of the surfaces of the crystal.

CHAP. III.

Description of a Goniometrical Telescope.

THE Klinometer, which we have described in Chap. I. may be successfully employed, when the line whose inclination is to be measured can be distinctly seen by the naked eye, and when a very accurate result is not required. But if the object is at a distance from the observer, or if the angle must be determined with accuracy, it will be necessary to make use of the goniometrical telescope.

This instrument is represented in Plate VI. Fig. 1., where TT is the eye-tube of the telescope, which carries the graduated circle AB, divided into 360 degrees. By means of the milled head which surrounds the eye-glass at E, this circle has a motion of rotation about the axis of the eye-tube. The vernier V has likewise a motion round the axis of the instrument, and may

be set to the zero of the scale, when the level, L, fixed to the plane surface of the graduated circle, is adjusted to a horizontal line. On the same surface, parallel to the axis of the level, there are fixed two screws, (one of them is seen at *s*,) on which the arm DF may slide to or from the eye-glass E. This arm is bent into a right angle a D, and carries a frame in which the small reflecting plane O, made of black glass, is fitted so as to have a rotatory motion about the axis *a b*.

When an angular object appears in the field of the telescope, the arm DF is pushed backwards or forwards, till the mirror O is near the centre of the eye-glass, and it is then turned round its axis *a b*, by means of the lever *h*, till the observer, by looking through the eye-glass, and into the mirror at the same time, perceives a distinct reflected image of the field of view, and the angular object which it contains. The graduated circle AB is then moved round its centre, till the reflected image of one of the lines which contains the angle is continuous with the line itself, and the degree pointed out by the index is noted down. The circle is again made to revolve till the image of the other line is continuous with the line itself, and the place of the index is again marked. The arch of the circle intercepted be-

tween these positions, is the measure of the angle required. To save the trouble of reading off a second line, the vernier may be placed at the zero of the scale, when the first coincidence has been observed.

In order to explain the theory of this instrument, let ABC, Plate VI. Fig. 2. be a plane angle seen in the field of the telescope, and MN the section of a reflecting mirror, which moves along with the graduated circle. When the side BC is in the same straight line with its image CE, BC is perpendicular to MN; and when, by the motion of the divided circle, the mirror MN is brought into a position *m n* perpendicular to the other side AB, the arch described by the moveable circle is evidently a measure of the angle formed by the lines AB, BC. The angular motion of the mirror, in passing from the position MN to *m n*, is not measured by the angle AOC formed at the centre O by AO and CO, but by the angle FOG, which is equal to ABC. This will be evident from considering, that the lines AB, CB are parallel to FO GO, and that the same angle would have been obtained by taking the reflected image of the lines FO and GO.

When the instrument is required to measure the apparent angle which any right line makes

with the horizon, the index of the vernier should point to zero when the level is adjusted to the horizon; and then, by turning round the graduated circle till the coincidence between the direct and reflected image of the right line is observed, the index will point out the angle required.

This instrument may, in some cases, be employed to measure an angle when it is not contained by right lines. If it is required, for example, to measure the apparent angle, which the distance BC, Plate V. Fig. 12., subtends at any other point O, we have only to make the axis round which the mirror *m n* moves point to O, and observe the two positions of the mirror, when the points B, C, seen by reflection, come in contact with any point D seen by direct vision, and the intercepted arch will be the angle at O. If one of the points B, C is at a greater distance from O than the other, it is evident that their reflected images cannot both come in contact with D. A motion of the mirror, however, in a plane at right angles to the plane in which it revolves, will produce the contact without affecting the result.

CHAP. IV.

Description of a Goniometrical Microscope.

THIS instrument, which we have represented in Plate VI. Fig. 3., is nothing more than the application of the contrivance described in the preceding Chapter to a simple or compound microscope. The graduated head AB, the arm FD, and the reflector O, are all constructed and used exactly in the same way as in the goniometrical microscope. The level h, however, is of no use in the present instrument, as the angles to be measured have no relation to a horizontal line. The object of this goniometer is to determine the angles of crystals, which are too minute to be applied to the common goniometer; and this is done by measuring the plane angles by which any solid angle of the crystal is contained, and then finding, by the formula given in a preceding Chapter, the angle of the planes themselves. The same result

may be obtained by measuring with this instrument the angle contained by the bounding lines of every section of a crystal, when it is reduced by projection to the angle contained by the planes. In this case, the plane of the graduated circle must be parallel to a plane passing through the bounding lines; an adjustment which can easily be effected by a contrivance for giving the crystal a motion in every direction.

There are numerous cases in microscopical observations, where it is necessary to measure very minute angles, when the plane passing through the lines by which the angle is contained is perpendicular to the eye; and there is no method but the one now described, by which this can be done with facility and accuracy. The goniometrical microscope, therefore, will have a very extensive application in the arts and sciences, and in many cases will be found of very essential advantage to the experimental philosopher.

CHAP. V.

Description of an Angular Wire Micrometer.

The most obvious method of measuring the angle contained by two lines, when the eye is above the plane which passes through them, is to have a pair of wires crossing one another in the centre of the field of a telescope, and one of them moveable, so as to form at the centre of the field every possible angle with the other. Dr Herschel's position micrometer is nothing more than an instrument of this kind. The moveable wire is turned round in the field of the telescope by a pinion, which works in the teeth of a wheel across which the wire is stretched. By this means the moveable wire forms every possible angle with the one which is fixed, and an index points out on a circular scale the angle which is contained by the wires.

Dr Herschel employed this instrument for mea-

suring the angle which a line, joining the two stars that compose a double star, forms with the line of their apparent motion. The micrometer was placed in such a position, that the larger star moved along the fixed wire, and the moveable wire was turned round till it passed through the two stars, S, s, Plate III. Fig. 2. The angle pointed out on the scale was the angle of position required. Dr Herschel suspected, that the smaller of the two stars which compose a double star revolved round the greater, or rather round their common centre of gravity; and, by means of this instrument, he found, that, in the double star of Castor, this revolution was performed in 342 years.

In this instrument the two wires always cross each other at the centre of the field, and consequently their angular separation is produced uniformly by the motion of the pinion; or the angular motion of the moveable wire is always proportional to the angular motion of the pinion. This very circumstance, however, which though it renders it easy for the observer to read off the angle from the scale, is one of the greatest imperfections of the instrument. The observations must obviously be all made on one side of the centre of the field, as appears from Plate III. Fig. 2., and the use of the instrument is limited to those cases in

which Ss is less than the radius SC. The greatest disadvantage of the instrument, however, is the shortness of the radius SC, for the error of observation must always diminish as the length of this radius increases. This disadvantage does not exist in measuring the angle of position of two stars S, s, for the distance Ss remains the same whatever be the length of SC; but in determining all other angles contained by lines, whose apparent length is greater than SC, this imperfection is inseparable from the instrument. Nay, there are some cases in which the instrument completely fails; as, for instance, when we wish to measure the angle formed by two lines which do not meet in a point, but only tend to a distant vertex. If the distance of the nearest extremities of these lines is greater than the chord of the angle which they form, measured upon the radius SC, then it is impossible to measure that angle, for the wires cannot be brought to coincide with the two lines by which it is contained. Nay, when the chord of the angle does exceed the distance between the nearest extremities, the portion of the wires that can be brought into coincidence with the lines is so small, as to lead to very serious errors in the result.

The new angular micrometer, which we ven-

ture to propose as a substitute for this instrument, is completely free from the defects which we have just noticed, and is founded on a very beautiful property of the circle. If any two chords AB, CD, Plate V. Fig. 13. intersect each other in the point O within the circle, the angle which they form at O will be equal to half the sum of the arches AC, BD; but if these chords do not intersect each other within the circle, but tend to any point O without the circle, as in Fig. 14., then the angle which they form is equal to half the difference of the arches AC, BD; that is, calling φ the angle, we have in the first case $\varphi=\frac{AC+BD}{2}$, and in the second case $\varphi=\frac{AC-BD}{2}$. Hence if AB, CD be two wires, placed in the focus of the first eye-glass of a telescope, the moveable one AB may be made to form every possible angle with the fixed one CD, and that angle may be readily found from the arches AB, CD.

The apparatus by which these arches are measured is represented in Plate IV. Fig. 3., and is nearly the same as that which is employed to measure the arches comprehended between the two steel points of the rotatory micrometer. In the present instrument, however, the graduated circular head might be divided only into 180°, in order to save the trou-

ble of halving the sum, or the difference of the arches AC, BD; but as it would still be necessary to measure *two* arches before the angle could be ascertained, we have adopted another method, remarkable for its simplicity, and giving no more trouble than if the wires always intersected each other in the centre of the field.

Let AB, for example, Plate V. Fig. 13., be the fixed wire, and CD the moveable one, and let it be required to find, at one observation, the angle AOC or φ. Let the index of the vernier be at zero, when the point D coincides with B; and as it is obvious that the extremity C will be at c when D is at B, the arch c A will be a constant quantity, which we shall call b. Making AC$=m$ and BD$=n$, we have, by the geometrical property already mentioned,

$$\varphi = \frac{m+n}{2};$$

but since the extremity C will move over the space Cc while D describes the space DB, these arches must be equal, consequently we have

$b = m - n$; hence adding $2\,n$ to each side of the equation, we obtain

$b + 2n = m + n$, and dividing by 2

$\frac{1}{2}\,b + n = \frac{m+n}{2}$, consequently

$\varphi = \frac{1}{2}\,b + n$

Hence the angle AOC is equal to half the arch A c added to the arch DB; or since A c is invariable, the half of it is a constant quantity, and the angle required is equal to the sum of this constant quantity and the arch DB.

When the wires do not intersect each other, as in Fig. 14., we have

$$\varphi = \frac{m - n}{2} \text{ and,}$$

$b = m + n$; hence subtracting $2\,n$ from each side of the equation, we have

$$b - 2\,n = m - n; \text{ and dividing by 2}$$

$$\tfrac{1}{2}b - n = \frac{m - n}{2} \text{ consequently}$$

$$\varphi = \tfrac{1}{2}b - n$$

That is, the angle AOB is equal to the difference between half the arch A c and the arch DB, or to a constant quantity, diminished by the arch DB.

In finding the angle AOB, therefore, we have merely to observe the place of the index when the wires are in their proper position; and as the scale commences at B, or when D and B coincide, and is numbered both ways from B, the degree pointed out on the circular head, when increased or diminished by the constant quantity, will give the angle of the wires which is sought. The semicircle on each side of a diameter drawn through

B, is divided into 180°, the 180th degree being at the opposite end of that diameter.

The method of reading off the angle AOB, may be still farther simplified, so as to save the trouble even of recollecting the constant quantity, and of adding and subtracting it from the arch pointed out by the index of the vernier. This effect is produced by making the index of the vernier point to the constant quantity upon the part of the scale below B, Fig. 13., when the points D, B coincide, or when the wire CD is in the position c B; for it is obvious that if z is the zero of the scale, and B z equal to the constant quantity, the arch D z, which is pointed out by the index of the vernier, will be equal to $\frac{1}{2}b + n$, or the angle AOB. In like manner in Fig. 14., where the wires do not cross each other within the field, and where B z is the constant quantity, the arch D z marked out by the index of the vernier, is obviously equal to $\frac{1}{2}b$—n, or the angle AOB, which the wires tend to form at O. By means of this adjustment, therefore, we are enabled to read off the angle AOB with the same facility as if the wires intersected each other in the very centre of the field, when the arches are accurate measures of the angles at the centre.

It is not necessary that the two wires should be placed in the focus of the first eye-glass. I have

constructed an instrument of this kind, in which the fixed wire AB is placed in the focus of the whole eye-piece, or, what is the same thing, in the focus of the object glass, while the moveable wire CD revolved in the focus of the first eye-glass. In this case the wire AB is more magnified than the other; but if this should be regarded as an inconvenience, it might easily be removed by using a more delicate fibre.

The graduated head upon which the scale of this instrument is engraven, is the same as that of the rotatory micrometer. The end of the eye-tube is represented in Plate IV. Fig. 3., where CD is the circular head, divided into 360°, and subdivided by the vernier V; L is the level, and AB the part of the eye-piece which contains the diaphragm with the fixed and moveable wires. The head CD, and the level L, are firmly fixed to the eye tube T, and from the head CD there rises an annular shoulder concentric with the tube; and containing the diaphragm across which the fixed wire is stretched, This diaphragm, which is represented in Fig. 4. with the wire extended across, projects through the circle of brass EF. All these parts remain immoveable, while the outer tube AB, and the other half EF of the circular head which contains the vernier V, have a rotatory motion upon the shoulder which rises from CD. The tube AB is merely an

outer case to protect a little tube within it, which contains the eye-glass, and the moveable diaphragm with its fibre extended across it. The inclosed tube is screwed into the ring EF, and the outer tube is also screwed upon the same ring; so that by moving AB, a motion of rotation is communicated to the vernier V, and to the diaphragm and wire belonging to the inner tube, while the rest of the eye-piece, containing the other diaphragm with its wire, remains stationary. By this means the moveable wire is made to form every possible angle with the fixed wire, and the angle is determined by the method which we have already explained. The fixed wire is placed a little out of the centre of the diaphragm to which it belongs, and the diaphragm itself is placed in a cell, in which it can be turned round, so as to adjust the wire to a horizontal line, when the level is set. The moveable wire is likewise placed at a little distance from the centre of its diaphragm, as represented in Fig. 5.; but by means of screws which pass through the inner tube into the edge of this diaphragm, it can be moved in a plane at right angles to the axis of the eye-piece, so that the moveable wire may be placed either in the centre of the field, or at different distances from it.

This instrument may be employed in microscopical observations.

CHAP. VI.

Description of a Double-Image Goniometer.

In every instrument in which a double image of an object is formed by means of two semilenses, with their centres at a distance, the one image appears to have a rotatory motion round the other when the telescope is turned about its axis. Thus in Plate VI. Fig. 4. if A, B be the images of two objects formed by the upper semilens when the common diameter of the semilenses is perpendicular to the horizon, and C, D, the images of the same objects, formed by the lower semilens; then by turning the telescope about its axis, or the semilenses round in their tube, the image A will appear to move round C in the circle A *a* E, and the image B round D in the circle B *b* F, or in the opposite directions if the telescope, or the semilenses are turned the other way. When the distance AC is equal to CD, as in Fig. 5. the image A will pass

over the image D; or if the telescope is turned in the opposite direction, the image B will pass over the image C. In like manner, when AC is greater or less than CD, the images will move as we have represented them in Fig. 4. and 6. In all these cases the four images may be brought into one straight line; and when this takes place, the line which passes through all the images will uniformly form the same angle with the horizon, as the common diameter of the semilenses. It is very easy to ascertain, with the utmost accuracy, when the images form one straight line; but particularly in the case where AC, Fig. 5. is equal to CD, for the image of A will then pass over D; and the coincidence of the images will mark the instant when the line which joins them is parallel to the common diameter of the lenses. Hence, as we obtain by this means the relation of the line joining the images to a fixed line in the instrument, the relation of this line to the horizon may be easily found by means of a level and a divided circular head. If the image is a straight line, then the coincidence of the two images, so as to form one straight line, will indicate the parallelism of that line to the diameter of the semilenses.

In constructing a goniometer of this kind sole-

ly for the purpose of measuring angles when the eye is not at their vertex, either the object-glass or the third eye-glass might be made the divided lens. If the object-glass is divided, it should be so constructed that it may have a rotatory motion in its cell, by applying the hand to a milled circumference AB, Plate VI. Fig. 7. Connected with the tube TT of the telescope is a circular ring of brass CD divided into 360°; and the divisions upon this scale are pointed out by the index of a vernier v, which moves along with the semilenses. A level L is fixed to the plate AB, having its axis parallel to the common diameter of the lenses, and being adjusted to a horizontal line when the index points to the zero of the scale. In using the instrument, therefore, the observer turns rounds the semilenses by means of the projecting milled circumference AB till the coincidence of the two images is distinctly perceived. The index of the vernier will then point out upon the graduated head the inclination of the line which is required.

When the telescope is long, this form of the instrument, though extremely simple, is not very convenient. The construction represented in Plate VII. Fig. 3. is in general to be preferred. This instrument consists of three tubes BL, LC,

CA. At the extremity B of the first tube is placed the divided object-glass, and at the other extremity L is fixed the divided circular head EF. The tube CL, which remains always at rest, is fixed to the stand HI by means of the clasp and screw at H. The tube AC, which contains an eye-piece, moves within both the tubes CL and LB. The tube BL extends towards C, within the tube CL, and round its circumference are cut a number of teeth in which the endless screw G works, and thus gives a rotatory motion to the tube LB, and the divided head EF. By this means the common diameter of the semilenses at B is made to form every possible angle with a horizontal line, which is indicated by a level above L, having its axis parallel to the common diameter of the semilenses. The index of the vernier scale *v*, fixed to the stationary tube CL, points out on the graduated head the angle required.

When the instrument is constructed with the third eye-glass divided instead of the object-glass, the graduated head and vernier must be placed upon the eye-tube, and made in the same way as for the rotatory micrometer.

If the principle of this goniometer is applied to the double-image telescope, which we have de-

scribed in Book I. Chap. III. and in Book III. Chap. II. and which consists of a divided lens moveable between the object-glass and its principal focus, we obtain an instrument of a very singular kind, which will measure at the same observation, the angle subtended at the eye of the observer by a line joining two points; and likewise the angle which that line forms with the horizon. When the two images of the line which joins the two points are brought into contact by the motion of the semilenses along the axis of the tube, these images must necessarily be in the same straight line; so that the relation of that line to the horizon, and the contact of the two images of it which determines its angular magnitude are obtained simultaneously, without any additional observation or adjustment. The one angle is read off on the rectilineal scale which extends along the tube, while the other is pointed out by the index of the vernier upon a circular divided head placed upon the same tube which carries the semilenses.

CHAP. VII.

Description of a Diagonal Telescope.

THE instruments which have already been described, with the exception of the double-image goniometer, can only be used with advantage when the angle to be measured is actually bounded by two right lines, or when two right lines are mutually inclined to each other, without meeting in a point. But when we have only two points in the lines, which is the case when we wish to measure the angle that a line joining two stars forms with the horizon, we are under the necessity of employing a different principle, for the eye cannot judge with any degree of accuracy when these points are situated in the same right line. In all the preceding instruments too, the radius of the graduated circle is necessarily very small, and the accuracy of the observation is obviously limited by the field of the telescope. In the following in-

strument all these disadvantages appear to be removed, and the principle of its construction seems to point out a method of measurement by which all angles of this kind may be determined with the utmost accuracy.

This instrument is represented in two different forms, in Plate VII. Fig. 4. and 5, where AB is a portion of a circle greater than a quadrant, and divided in the usual manner into degrees and parts of a degree. This graduated limb may be supported in different ways, according to the purposes to which the instrument is to be applied. A frame EF, attached to a moveable radius, moves round the centre C, by means of a clank and screw of the common construction, and it has a sufficient opening to admit a telescope DG, which can be placed in any position between a horizontal and a vertical line, by a motion round the pivots m, n, in a plane perpendicular to that of the semicircle. One of the extremities of the arm CV carries a vernier V for subdividing the degrees of the limb AB. In the focus of the telescope are placed three or four fibres parallel to each other, and perpendicular to the axis $m\,n$, and these are crossed by a horizontal fibre which passes through the centre of the field. When the instrument has been adjusted to a horizontal plane by the le-

vels attached to it, the arm CV, which carries the télescope, is raised into such a position, that during the motion of the telescope round the centres *m, n,* the intersection of the wires traces the line whose inclination to the horizon is required. When this position is obtained, the index will point out the angle upon the graduated semicircle. If only two points in the line are given, the arm CV is shifted along the limb, till the intèrsection of the wires passes through the two points.

This instrument may be employed with great advantage in measuring the angle which a line, joining two stars, forms with the horizon, or with a line joining other two stars; and hence it may be used for determining the hour of the night, and for finding the place of a comet, or any other celestial body. The line which joins any two stars, forms every possible angle with the horizon in the course of 23 hours and 56 minutes; so that by knowing the hour of the day when this angle is of any given magnitude, the hours corresponding to other angles may be obtained by simple proportion. The result which is thus obtained will of course require to be corrected by refraction. The diagonal telescope may also be employed in surveying, and for other important purposes.

CHAP. VIII.

Description of a New Protractor for laying down and measuring Angles upon Paper.

THE common protractor which is employed in laying down and measuring angles in trigonometrical plans, can only be used when the lines that contain the angle actually intersect each other. The centre of the protractor is laid upon the intersection of the lines, and the arch upon its circumference, intercepted between the lines, is a measure of their mutual inclination. When the lines, however, do not meet in a point, the angle which they tend to form cannot be ascertained without the additional operation of drawing a third line parallel to either of the other two. But even in the case where the point of intersection is given, it is often difficult, particularly when the angle is small, to find the exact point; so that, on this account, the risk of error is in-

creased. These remarks are equally applicable to the laying down of angles by the protractor.

In order to remedy these inconveniences, we propose to employ the geometrical principle which has already been explained in the fifth Chapter of this Book. Thus in Plate V. Figs. 13, 14, if the lines AB, CD form an angle at O, the protractor, whose divided circumference is represented by the circle ACBD, may be laid down in any way upon the lines AB, CD, so that the arches AC, BD intercepted between them may be distinctly observed. When the lines intersect each other in a point within the protractor, then half the sum of the arches will be the angle at O, and when the point O is without the protractor, the angle at O is equal to half the difference of these arches. In order to save the trouble of dividing by two, the circumference of the protractor is divided into 180° instead of 360°. If the protractor is furnished with points situated at the extremities of a moveable arm, projecting beyond the divided circumference, and carried round by a rack and pinion, the arches may be measured with great facility. Thus, in Plate VII. Fig. 2. when the point O is within the protractor, let the instrument be placed upon the lines AB, CD, so that the points P, *p*, at the ends of

the arms MP, M p may touch the lines AO, DO when the index of the vernier stands at the zero of the scale. Then turn the pinion till the point P comes to Q in the line CD. The other point p will be at q, when $p\,q = \text{PQ}$. Hence the arch $\frac{q\,r}{2}$ will be equal to the angle AOC; or, since the circumference is divided into 180° instead of 360°, the arch $q\,r = \text{AOC}$; so that by observing the place of the vernier when the point is at q, and by turning the pinion till the point comes to r, and again observing the position of the vernier, we obtain the arch $q\,r$, or the angle AOC. The same result might be obtained in a still simpler way, by placing one of the points at O and the other on the line OD, when the index of the vernier is at zero. If the pinion is now turned till the other point comes to the line OB, the index will point out upon the scale the angle AOC.

When the two lines do not meet, as in Fig. 1. the point P must be placed on the line AB, while the other point p is in the line CD. By moving the pinion till p comes to q in the line AB, the point P will have described the arch $\text{PQ} = p\,q$, so that $\text{Q}\,r = \text{PQ} - p\,q =$ the angle formed by the lines AB, CD. Hence, by making the point P describe the arch Qr, we obtain the angle required.

BOOK III.

ON INSTRUMENTS FOR MEASURING DISTANCES.

CHAP. I.

Description of a Telescope for measuring Distances.

In the first Chapter of Book I. we have already explained the principle upon which this instrument is constructed, and have shewn its application as a micrometer to the purposes of practical astronomy. We shall now proceed to point out the advantages with which the same principle may be extended to a portable telescope for measuring distances; and to describe the construction and use of the instrument, with as much

fulness and perspicuity as the nature of the subject will allow.

In ascertaining distances from the angles subtended by any object seen through a telescope, the accuracy of the result is obviously limited by the field of view. If the angle is very small, any error in the instrument, or in the observation, will produce a great error in the distance which is deduced from it; and this error will always diminish as the angle is increased. We obtain no remedy for this evil by augmenting the magnifying power of the instrument; for though the accuracy of the observation is increased, yet, as the angle has suffered a diminution exactly corresponding to the augmentation of power, the error is as much increased by the one cause as it was diminished by the other. In cases, therefore, where very great accuracy is required, the use of the telescope cannot be recommended; but in the majority of cases in which the distances of objects are required, this extreme accuracy is never wanted; and hence the micrometrical telescope will be found of very great utility to travellers, and to officers of the army and navy, who require an instrument that is portable, and not easily injured, while it can be employed with

facility, and without the trouble of tedious calculation.

The instrument which we are about to describe seems to possess these qualities in a very great degree, and at the same time combines the properties of a telescope, a microscope, and a trigonometrical instrument. As a telescope, it has a variable magnifying power; as a compound microscope, its power and distinctness are very considerable; and as a trigonometrical instrument, it may be employed under circumstances in which other instruments could not be conveniently used.

The micrometrical telescope is represented in its most general form in Plate VIII. Fig. 1., where AB, BC, CD, DE are the tubes of which it is composed. The principal object-glass is placed at A; the second or moveable object-glass is situated a little to the left of C, at the end of the tube DC; and the eyepiece is contained in the tube DE. When the tubes BC, CD are pushed into the outer tube AB, the moveable object-glass near C will be in contact with the principal object-glass at A; and in this situation distinct vision may be procured by pulling out the eyepiece DE: The magnifying power of the instrument is now a *minimum*. By pulling out the tube CD, the moveable object-glass near C is separated from the principal ob-

ject-glass, distinct vision may again be procured by pushing in the eye-tube DE, and the magnifying power of the telescope will be greater than before. In like manner, by pulling out the tube CB, and producing distinct vision by a new adjustment of the eyepiece, the distance of the two object-glasses, and likewise the magnifying power of the instrument, will increase, till the place of the moveable lens coincides with the principal focus of the object-glass, when the magnifying power of the telescope will be a *maximum*.

In this form of the instrument, the motion of the second object-glass, and the adjustment of the eye-tube to distinct vision, are produced merely by pushing in or drawing out the tubes; but it would be much more convenient for the observer if there were only two tubes between A and D, and if the motion of the tube which contains the second object-glass, and of that which contains the eyepiece, was effected by means of a rack and pinion. By this means the adjustments would all be produced with the utmost facility and accuracy; and by turning the pinion for the object-glass with the one hand, and the pinion for the eye-tube with the other, the observations would be made in much less time than if the tubes were shifted merely with the hand. As

telescopes, however, are so generally constructed with two or three moveable tubes as shewn in Plate VIII. Fig. 1., we shall accommodate our description to that form of the instrument.

The diaphragm or field-bar of the telescope, which is placed in the anterior focus of the first eye-glass, is represented in Fig. 2. where a, b are two delicate steel points projecting into the field of view, and $m\,n$, $o\,p$ a pair of parallel fibres, all of which are firmly fixed in their respective positions. For the purpose of measuring distances, the two steel points only are required: The wires may be used, indeed, for the same purpose; but they are principally intended for measuring the angle subtended by any body that is in motion. When it is required that the instrument should measure a great variety of angles, another steel point c will be of great advantage, and its position may be determined in the following manner. Let us suppose, that the greatest and least magnifying power of the telescope, or rather that the greatest and least angles subtended by the points a, b, are to one other as 3° to 1°, or as 180′ to 60′; then the instrument, by means of these two points, will only measure angles that are between 180′ and 60′. In order, therefore, to render it capable of determin-

ing angles less than 60′, another point *c* should be so placed in the field, that the chord of *a c* is to that of *a b* as 60′ to 180′; and consequently by using the points *a*, *c*, all angles between 60′ and 20′ may be measured. Another point might likewise be used if necessary, and the same method might be adopted with respect to the wires.

In fixing the diaphragm in its proper position, great attention must be paid to the rules laid down in Book I. Chap. IX. and in order that the adjustment of the diaphragm may be effected with the greatest facility and accuracy, it should be so constructed as to move by means of a screw along the axis of the eye-tube.

The next point to be considered, is the nature and construction of the scale, by which the variations of the angle are to be measured. This scale is engraven on the two moveable tubes BC, CD, and its length is equal to the focal distance of the principal object-glass, or, in general, to the space through which the second object-glass is allowed to move along the axis of the tube. When this object-glass is separated from the principal object-glass till their distance is equal to the focal length of the latter, the angle subtended by a pair of fixed points or wires will re-

main the same whatever be the focal length of the former, and will suffer no change even if the second object-glass be removed. In these circumstances, the magnifying power of the telescope is a *maximum*, and consequently the angle subtended by the points or wires is a *minimum*. If the second object-glass is moved from this position towards the principal object-glass, the *minimum* angle of the points or wires will be increased, and the increment which it receives will be directly as the space through which the second object-glass has been moved, and inversely as its focal length. Hence the extent of the variation which can be produced upon the *minimum* angle will be a *maximum* when the second object-glass is in contact with the principal object-glass, the focal length of the former remaining the same: And the extent of this variation may be increased at pleasure, by taking a moveable object-glass of a shorter focal length.

If the focal length of the principal object-glass, for example, is 30 inches, and that of the moveable one 50 inches, while the *minimum* angle of the wire is 40′, then, by the formula

$\alpha = A + \frac{Ab}{F}$, (See p. 15.) we have for the great-

est angle when the two lenses are in contact, and consequently when $b = 30$,

$$\alpha = 40' + \frac{40' \times 30}{50} = 64',$$ so that the greatest extent of the scale will in this case be $64' - 40 = 24'$.

If F, or the focal length of the moveable object-glass, is only two inches, then we have

$$\alpha = 40' + \frac{40' \times 30}{2} = 640' = 10^\circ 40'.$$

Hence the greatest extent of the scale will now be $10^\circ 40' - 40' = 10^\circ$.

As the length of the scale is 30 inches, one minute on the scale will be equal to 1.25 inches, when the greatest extent is 24′; but, when the greatest extent is 10°, one minute of the scale will be equal only to 0.05 inches. Hence it appears that, by diminishing the extent of the scale, we increase its magnitude, and consequently its accuracy, to such an unnecessary degree, that the minuteness of the scale far exceeds the accuracy of the observation; while, by increasing the extent of the scale, we diminish its magnitude and its accuracy, till it is incapable of affording us a measure of the angle corresponding with the excellence of the telescope. The moveable object-glass, therefore, should have such a focal length,

that the probable error of the scale is less than the error of observation; but, for common purposes, it will be sufficient to make the focal length of the moveable object glass equal to between one half and one third of that of the principal object glass.

The focal length of the second object glass and the extent of the scale being thus determined, the formation of the scale itself is the next subject of consideration. We have already demonstrated, that the motion of the moveable object glass, along equal portions of the axis of the telescope, corresponds to equal changes upon the *minimum* angle of the points or wires; and consequently, that the scale which measures these angular variations is a scale of equal parts. By determining, therefore, the *minimum* angle, or the angle subtended by the points, when the object glasses are at their greatest distance, and also the angle which they subtend when the two object glasses are in contact, we obtain the value of the angle at the two extreme points of the scale, and, consequently, its value at any intermediate point.

Thus, if the length of the scale is 30 inches, the *minimum* angle 40′, and the angle of the other extremity of the scale 120′, we have 60′ for the extent of the scale, so that half an inch on the scale

will correspond with 1′, and one tenth of an inch will correspond with 12″. The scale may, therefore, be divided into 60 equal parts, or halves of an inch, to shew minutes; into 300 equal parts, or tenths of an inch, to shew every 12″; or into 1200 equal parts, or fortieths of an inch, to shew every 3″. When the scale is engraven on more than one tube, as in Plate VIII. Fig. 1., the extremity C of the tube BC is the index for the divisions on the tube CD when CB is completely pushed into AB; and the extremity B of the tube AB is the index for the divisions on BC, when CD is completely drawn out. In measuring the distance between the two extremities B, D of the scale, the breadth of the milled circumference, at the extremity C of the tube BC, must evidently be omitted. The method of finding the angles, at the two extremities of the scale, by direct experiment, has already been fully explained in Book I. Chap. I. As the exact determination of these extreme angles is of the utmost consequence to the accuracy of the scale, the greatest care should be taken in conducting the observations.

Having thus pointed out the method of constructing the scale of the telescope, we shall now proceed to shew its application to the measurement of distances. Let D, E, Plate VIII. Fig. 3. be two

prominent points in the object whose distance is to be measured, and so chosen that at the station B the pair of points may be made to comprehend the space DE, without separating the two object glasses more than four or five inches. The line joining the points D, E may be either vertical, or horizontal, or inclined to the horizon. The points themselves should always be so distinct, that they can be recognised at the second station at A, and points of this kind may easily be found, whatever be the object to which the telescope is directed. The interval between two windows or any two projecting parts in a building, the distance between two stones lying in a field, or upon a hill, and the space between two trees, may be used for the purpose of measuring their distance from the observer.

The telescope being directed to the object, and the tubes BC, CD being pushed into the tube AB, and distinct vision procured by the adjustment of the eye tube DE, the two steel points should either exactly coincide with the two points D, E, Fig. 3. in the object whose distance is to be measured, or should occupy a greater space. If the steel points comprehend a greater interval than that which lies between the points D, E of the object, which is most likely to be the case, separate the object glasses by pulling out the first di-

vided tube CD, Fig. 1., and produce distinct vision, by the eye-piece DE, till the steel points exactly coincide with the points D, E, Fig. 3., or comprehend the object DE, the extremity C of the drawer CD (Fig. 1.,) will then mark out upon the scale the value of the angle DBE. Having measured a base BA, reckoned always from the object glass of the telescope, and equal to about $\frac{1}{7}$ or $\frac{1}{8}$, or a greater part of the whole distance, place the object glass of the telescope at A, and again direct it to the object DE. The distance DE will now subtend a much less angle at A than it did at B, and hence the steel points will comprehend a much greater space than DE. In order to shut the steel points, therefore, pull out the tube CD; or, if necessary, part of the tube BC also, and adjust the eye tube to distinct vision, till the steel points exactly coincide with the points D, E of the object; then, if the tube CD has not been completely drawn out, the extremity C will mark upon the scale between C and D the angle DAE; or, if the tube CD is wholly drawn out, and likewise part of the tube CD, then the extremity B of the outer tube AB will mark upon the scale, between B and C, the value of the angle DAE. If a vernier were placed at the extremities B and C of the tubes AB and BC, the angles might be read off with the

greatest accuracy; and all the observations would be greatly facilitated if the tubes were moved by a rack and pinion.

Having now obtained the angles DBE, DAE, or rather the tangents of these angles, (to which the angles themselves are nearly proportional,) let us call DBE = m, DAE = n, and AB = a. Then, since AC is to BC, as the tangent of the angle at B is to the tangent of the angle at A, that is, as m is to n, we have obviously the ratio of the two distances AC, BC, and the difference AB of these distances; so that AC or BC may be found by one of the simplest theorems in Algebra, namely, to find the value of two numbers, whose ratio and difference are given.

This theorem gives us

$$AC = \frac{a\,m}{m-n}, \text{ and } BC = \frac{a\,n}{m-n}$$

Let us suppose, for example, that the angle DBE, or m, is equal to 68 minutes, and DAE, or n, equal to 46 minutes, and that the base AB, or a, is equal to 120 feet, then we have

$$AC = \frac{120 \times 68}{68-46} = \frac{8160}{22} = 370\tfrac{10}{11} \text{ feet.}$$

$$BC = \frac{120 \times 46}{68-46} = \frac{5520}{22} = 250\tfrac{10}{11} \text{ feet.}$$

Hence we obtain the following rules:

For the greater distance AC. *Multiply the length*

of the base in yards, or feet, by the greatest angle; divide the product by the difference between the two angles, and the quotient will be the distance required in yards or feet.

For the lesser distance BC. *Multiply the length of the base in yards, or feet, by the least angle; divide the product by the difference between the angles, and the quotient will be the distance required in yards or feet.*

It is manifest from the preceding observations, that there is no necessity of having the real angles subtended at A and B, by the object DE, the ratio of the angles being all that is wanted in practice. On this account the instrument is sometimes constructed with only the ratios of the angles engraven upon the scale. In this case the angles themselves are easily found, when either the *maximum* or *minimum* angle is determined.

The micrometrical telescope possesses one very singular property, to which we would request the particular attention of the reader. If any other micrometer had been employed to measure the distance of the object DE, by taking the angles which it subtends at B and A, it would have been necessary to apply a correction to the angle, arising from the aberration in the focal length of the telescope, at the different distances AC, BC.

This aberration, or increase, in the focal length, produces a corresponding diminution in the angle, so that we should have been obliged to diminish each angle in the ratio of $F + \frac{F^2}{D-F}$ or $\frac{DF}{D-F} : F$,* where F is the principal focal length of the telescope, and D the distance of the object; but as this very correction supposes the distance of the object to be known, it would be necessary to find the distance, upon the supposition that no correction was requisite, and then to have applied the correction in the preceding formula, computed from the approximate distance.

In the micrometrical telescope, however, no such correction is necessary. The virtual focal length of the combined object glasses, when the steel points comprehend the angle DBE, is to their virtual focal length, when the points comprehend the angle DAE, in the very same ratio as the angles themselves; and consequently the corrections, which are as the focal lengths, will also have the same ratio as the angles. These corrections, therefore, will not alter the ratio of the angles found by the instrument, and hence the application of them is unnecessary.

Thus, let F, f be the principal virtual focal

* See Book I. Chap. I. p. 20.

lengths of the combined glasses, at the stations A and B respectively. Make AC = D, and BC = d; Then, since the magnifying powers are directly as the conjugate focal lengths, since the angles are inversely as the magnifying powers, and the angles inversely as the distances AC, BC, the conjugate focal lengths will be directly as the distances, that is,

$$\frac{DF}{D-F} : \frac{df}{d-f} = D : d.$$

To explain this numerically,
make $D = 200$, $d = 100$, $F = 12$, $f = 6$, we shall have $\frac{DF}{D-F} = \frac{2400}{200-12} = 12{,}76595$, and

$$\frac{df}{d-f} = \frac{600}{100-6} = 6.38297. \text{ But}$$

$12.76595 : 6.38297 = 12 : 6 = 200 : 100$; so that whatever be the distance of the object, the ratio of the angles obtained by the micrometrical telescope require no correction, in order to obtain an accurate result.

It is obvious, however, that when we wish to measure with this instrument the absolute value of the angles, these values must be corrected according to the distance of the object, by means of the formula given in page 20. In cases where the object is at a considerable distance, the incre-

ment of the angle, as found by the formula, is too trifling to be taken into account, unless where very great accuracy is required.

There is one application of the micrometrical telescope to the mensuration of distances, which may, on many occasions, be of considerable service to the military engineer, namely, to measure the distance of an object from a station where the inequalities of the ground render it impossible to procure a base; and it does this by a single observation, which is of the greatest consequence in particular cases, where the proximity of the enemy's guns makes it difficult to measure two angles and a base with deliberation and safety.

Let it be required, for example, to measure the distance MC, Plate VIII. Fig. 4. from a station M where works are to be erected, within reach of the enemy's guns, at C, and where the inequalities of the ground prevent the mensuration of a base. Take two stations A, B beyond the reach of the enemy's guns, and in the same vertical plane with M, and by measuring the base AB, and ascertaining, according to the method already explained, the angles aBb, aAb subtended by any object ab,* at

* It is not necessary that the line ab should be perpendicular to the axis of the telescope. It is only requisite that it should be equally inclined to that axis at the stations A, B, and M,

C find the distance AC. Then from the station M, measure the angle a M b, subtended by the same object $a\, b$ at M, and the distance MC will be a fourth, proportional to the angle a M b, the angle a A b, and the distance AC; that is, if the angle $a\mathrm{M}b=m$, $a\mathrm{A}b=n$, we shall have $m : n = \mathrm{AC} : \mathrm{MC}$, and $\mathrm{MC} = \frac{\mathrm{AC} \times n}{m}$, which gives the following rule:

The lesser distance is equal to the greater distance, multiplied by the angle which corresponds to the greater distance, and divided by the angle which corresponds to the lesser distance.

By means of this instrument, we may also measure accessible and inaccessible heights with great facility and accuracy. Thus, let BA, Plate VIII. Fig. 5., be an accessible height, then, from a point B, as near as possible to the vertical line BA, measure the angle a B b, subtended by any object $a\, b$ at the top of the height, and call this m. Measure also the angle a C b, subtended by the same object at C, which call n, and then find the length of the base BC, which call a. Hence,

$$\mathrm{AC} : \mathrm{AB} = m : n, \text{ and}$$

$$\mathrm{AB} = \frac{\mathrm{AC} \times n}{m}. \text{ But by Euclid, 47. I.}$$

which will be the case when these points are in the same vertical plane.

$AC^2 = a^2 + \frac{AC^2 \times n^2}{m^2}$ multiplying by m^2, and transposing, we have

$$AC = \sqrt{\frac{m^2 a^2}{n^2 - m^2}}, \text{ or}$$

$$AC = \frac{m a}{\sqrt{n^2 - m^2}}$$

By similar reasoning we obtain

$$AB = \frac{n a}{\sqrt{m^2 - n^2}}$$

From these formulæ we derive the following rules:

For the perpendicular height. *Multiply the greater angle by the base, and divide this product by the square root of the difference between the squares of the greater and the lesser angles; the quotient will be the height required.*

For the hypothenuse. *Multiply the lesser angle by the base, and divide this product by the square root of the difference between the squares of the greater and the lesser angles; the quotient will be the length of the hypothenuse required.*

When the height AB is inaccessible, find the angle subtended by an object $a\ b$ at the stations C and D, and, having measured the base CD, find, by the method already described, the distance

CB. Then, if n represents the angle taken at C, and m that taken at D, we have

$$AD : AC = n : m, \text{ and}$$

$$AD = \frac{AC \times n}{m}$$

But by Playfair's Euclid, Book II. Prop. 12., we have

$$AD^2 = AC^2 + CD^2 + \overline{2DC \times CB}, \text{ and}$$

$$AD^2 - AC^2 = CD^2 + \overline{2DC \times CB}.$$

By substituting the preceding value of AD, we obtain

$$\frac{AC^2 \times n^2}{m^2} - AC^2 = CD^2 + \overline{2DC \times CB}, \text{ and by}$$

reduction,

$$AC = \sqrt{\frac{CD^2 + 2\overline{DC \times CB}}{\frac{n}{m^2} - 1}} \text{ or,}$$

$$AC = \sqrt{\frac{\overline{BD^2} - \overline{BC^2}}{\frac{n^2}{m^2} - 1}}$$

The preceding method is applicable only to inaccessible heights, where the distance CB is capable of being measured by the instrument.

As the angle subtended by the steel points, when the two object glasses are in contact, is very considerable, we may by successive measurements determine angles of any magnitude. In Plate VIII. Fig. 6., for example, the angle ACB may be found by measuring successively the angles ACe, eCf,

fCB, and, in the case of a mountain where the vertical line AB is not visible, the angle ACB may be ascertained by finding in succession the angles ACe', e'Cf', f'Cb'. For this purpose, the telescope should have a level fixed upon the eye-tube, with its axis parallel to the axis of the instrument, so that the lower steel point might cover a point b' in the horizontal line BC, when the level is adjusted.

Having now shewn, at considerable length, the method of using the micrometrical telescope, when we are completely unacquainted with the magnitude of the object whose distance is to be measured, we shall next proceed to point out its use in determining distances, when the magnitude of the object or of any part of it is known.

Let DE, Plate VIII. Fig. 4. be a distant object seen by the observer at B, and perpendicular to the line BC, then, if the length of DE be known, and if the angle DBE be measured with the micrometrical telescope, the distance BC may be found by the simplest case of plain Trigonometry, which gives

$$BC = \frac{\text{Cot. } \frac{1}{2} DBE \times \frac{1}{2} DE}{\text{Rad.}}$$, or since the angle is very small, we may use the formula

$$BC = \frac{\text{Cot. } DBE \times DE}{\text{Rad.}}$$

That is, *add the logarithm cotangent of the angle to the logarithm of the length of the object, and the sum, subtracting radius, will be the logarithm of the distance required.*

In order, however, to save the trouble of trigonometrical calculation, we have computed a series of tables, which are given at the end of the Chapter, shewing the distance of a given object, when it subtends different angles at the eye of the observer. The first table is calculated for an object one foot in length; the second for an object six feet in length; the third for an object 20 feet in length; the fourth for an object 30 feet in length; and the fifth for an object 40 feet in length.

By means of these tables, the distance of an object may be ascertained by mere inspection when the angle is found in minutes, and when the magnitude of the object is, 1, 6, 20, 30, and 40 feet. If the angle is found both in minutes and seconds, and if the magnitude of the known object is neither 1, 6, 20, 30, or 40 feet, the distance may be determined by simple proportion. Thus:

EXAMPLE I. If the object whose distance is required is 6 feet, which is nearly the height of a man, and subtends an angle of 49 minutes, then, by looking into Table II. opposite 49 minutes.

we find 421 feet, which is the distance of the object. If the angle be 49′ 20″, then the distance of the object will be between 421 and 412½ feet; and the exact number will be found by simple proportion, thus:

Feet. Feet.
60″ : 8.5 = 20″ : 2.8, which being subtracted from 421, leaves 418.3 feet.

EXAMPLE II. If the object is 20 feet, and the angle which it subtends 18′ 16″; then in Table IV. opposite 18′, we have 3820 feet; and opposite 19′, 3619 feet, the difference of which is 201 feet. Hence we have

Feet. Feet.
60″ : 201 = 16″ : 53.6, which, subtracted from 3820, gives 3766 for, the distance of the object.

EXAMPLE III. If the length of the object is neither 1 foot, 6, 20, 30, or 40, but any other number, such as 5, and the angle 50′, then enter Table I. with the angle, and opposite 50 will be found 68.7, which, multiplied by 5, the real length of the object, gives 343.5 for, the distance required.

The same answer might be obtained by taking the 4th part of the result given by Table III., the 6th part of the result given in Table IV., and the 8th part of the result given in Table V.; since 5

is the 4th part of 20, the 6th part of 30, and the 8th part of 40. Thus, opposite to 50′, in these tables, we have 1375, 2062, 2750, which, being divided by 4, 6, and 8 respectively, leaves 343 feet.

The third, fourth, and fifth tables may be particularly useful in taking a rapid survey of a country by means of the micrometrical telescope. Two objects placed at the extremities of a chain, 20, 30, or 40 feet long, may be placed opposite to the telescope by one person, while another measures the angle which they subtend, and thus obtains a succession of distances with the greatest facility.

Sir George Mackenzie carried along with him to Iceland one of the micrometrical and double-image telescopes, and a copy of the following Tables, in order to make a general survey of the part of the island which he visited; but the difficulties which he had to encounter in the carriage of his instruments, and from the ruggedness of the ground, prevented him from doing the same service to the geography which he has done to the geology and mineralogy of that interesting country.

TABLES

SHEWING THE

DISTANCES AT WHICH DIFFERENT ANGLES ARE SUBTENDED

BY

BODIES OF DIFFERENT LENGTHS.

TABLE I. *Shewing the Distances at which different Angles are subtended by a Body* ONE *Foot in Length.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
1	3437.7	32	107.4	63	54.5
2	1718.9	33	104.2	64	53.7
3	1146.0	34	101.1	65	52 9
4	859.4	35	98.2	66	52.1
5	687.6	36	95.5	67	51.3
6	573.0	37	92.9	68	50.5
7	491.0	38	90.5	69	49.8
8	429.7	39	88.1	70	49.1
9	382.0	40	85.9	71	48.4
10	343.8	41	83.8	72	47.7
11	312.5	42	81.8	73	47.1
12	286.5	43	79.9	74	46.4
13	264.4	44	78.1	75	45.8
14	245.5	45	76.4	76	45.2
15	229.2	46	74.7	77	44.6
16	214.9	47	73.1	78	44.1
17	202.2	48	71.6	79	43.5
18	191.0	49	70.2	80	43.0
19	180.9	50	68.7	81	42.4
20	171.9	51	67.4	82	41.9
21	163.7	52	66.1	83	41.4
22	156.3	53	64.9	84	40.9
23	149.5	54	63.7	85	40.4
24	143.2	55	62.5	86	40.0
25	137.5	56	61.4	87	39.5
26	132.2	57	60.3	88	39.1
27	127.3	58	59.3	89	38.6
28	122.7	59	58.3	90	38.2
29	118.5	60	57.3	91	37.8
30	114.6	61	56.4	92	37.4
31	110.9	62	55.4	93	37.0

TABLE I.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
94	36.6	129	26.6	164	21.0
95	36.2	130	26.4	165	20.9
96	35.8	131	26.2	166	20.7
97	35.4	132	26.0	167	20.6
98	35.1	133	25.8	168	20.5
99	34.7	134	25.6	169	20.3
100	34.4	135	25.4	170	20.2
101	34.0	136	25.2	171	20.1
102	33.7	137	24.0	172	20.0
103	33.4	138	24.2	173	19.9
104	33.0	139	24.6	174	19.8
105	32.7	140	24.5	175	19.7
106	32.4	141	24.4	176	19.5
107	32.1	142	24.2	177	19.4
108	31.8	143	24.0	178	19.3
109	31.5	144	23.9	179	19.2
110	31.2	145	23.7	180	19.1
111	31.0	146	23.5	181	19.0
112	30.7	147	23.3	182	18.9
113	30.4	148	23.2	183	18.8
114	30.1	149	23.0	184	18.7
115	29.9	150	22.9	185	18.6
116	29.6	151	22.7	186	18.5
117	29.4	152	22.6	187	18.4
118	29.1	153	22.5	188	18.3
119	28.9	154	22.3	189	18.2
120	28.6	155	22.2	190	18.1
121	28.4	156	22.1	191	18.0
122	28.2	157	21.9	192	17.9
123	27.9	158	21.8	193	17.8
124	27.7	159	21.6	194	17.7
125	27.5	160	21.5	195	17.7
126	27.3	161	21.4	196	17.6
127	27.1	162	21.3	197	17.5
128	26.8	163	21.1	198	17.4

TABLE I.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
199	17.3	223	15.4	247	13.9
200	17.2	224	15.4	248	13.9
201	17.1	225	15.3	249	13.8
202	16.0	226	15.2	250	13.7
203	16.9	227	15.2	251	13.7
204	16.9	228	15.1	252	13.6
205	16.8	229	15.0	253	13.6
206	16.7	230	14.9	254	13 5
207	16.6	231	14.9	255	13.5
208	16.5	232	14.8	256	13.4
209	16.5	233	14.8	257	13.3
210	16.4	234	14.7	258	13.3
211	16.3	235	14.7	259	13.2
212	16.2	236	14.6	260	13.2
213	16.1	237	14.5	261	13.1
214	16.1	238	14.4	262	13.1
215	16.0	239	14.4	263	13.0
216	15.9	240	14.3	264	13.0
217	15.9	241	14.3	265	13.0
218	15.8	242	14.2	266	12.9
219	15.7	243	14.1	267	12.8
220	15.6	244	14.1	268	12.8
221	15.6	245	14.0	269	12.7
222	15.5	246	14.0	270	12.7

TABLE II. *Shewing the Distances at which different Angles are subtended by a Body* SIX *feet long.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
1	20626.8	32	644.5	63	327.6
2	10313.	33	625.	64	322.5
3	6875.4	34	606.6	65	317.5
4	5156.	35	589.3	66	312.5
5	4125.2	36	572.9	67	307.9
6	3437.7	37	557.5	68	303.3
7	2946.6	38	542.8	69	299.0
8	2578.2	39	528.9	70	294.7
9	2291.8	40	515.6	71	290.5
10	2062.6	41	503.1	72	286.4
11	1875.2	42	491.1	73	282.5
12	1718.8	43	479.7	74	278.7
13	1586.7	44	468.8	75	275.0
14	1473.3	45	458.4	76	271.4
15	1375.0	46	448.4	77	267.9
16	1298.1	47	438.9	78	264.4
17	1213.3	48	429.7	79	261.1
18	1145.9	49	421.	80	257.8
19	1085.6	50	412.5	81	254.7
20	1031.4	51	404.4	82	251.5
21	982.2	52	396.7	83	248.5
22	937.6	53	389.2	84	245.5
23	896.8	54	381.9	85	242.7
24	859.4	55	375.	86	239.8
25	825.	56	368.3	87	237.1
26	793.3	57	361.9	88	234.4
27	763.9	58	355.6	89	231.8
28	736.6	59	349.6	90	229.2
29	711.3	60	343.8	91	226.7
30	687.5	61	338.2	92	224.2
31	665.4	62	332.7	93	221.8

TABLE II.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
94	219.4	129	160.0	164	125.7
95	217.1	130	158.7	165	125.0
96	214.8	131	157.5	166	124.2
97	212.7	132	156.2	167	123.5
98	210.5	133	155.1	168	122.7
99	208.4	134	153.9	169	122.0
100	206.3	135	152.8	170	121.3
101	204.2	136	151.6	171	120.6
102	202.2	137	150.6	172	119.9
103	200.3	138	149.5	173	119.2
104	198.3	139	148.4	174	118.5
105	196.5	140	147.3	175	117.9
106	194.6	141	146.3	176	117.2
107	192.9	142	145.2	177	116.6
108	190.9	143	144.2	178	115.9
109	189.2	144	143.2	179	115.3
110	187.5	145	142.2	180	114.6
111	185.8	146	141.2	181	114.0
112	184.1	147	140.3	182	113.3
113	182.5	148	139.3	183	112.7
114	180.9	149	138.4	184	112.1
115	179.4	150	137.5	185	111.5
116	177.8	151	136.6	186	110.9
117	176.3	152	135.7	187	110.3
118	174.8	153	134.8	188	109.7
119	173.4	154	133.9	189	109.1
120	171.9	155	133.1	190	108.5
121	170.5	156	132.2	191	107.9
122	169.1	157	131.4	192	107.4
123	167.7	158	130.5	193	106.8
124	166.3	159	129.7	194	106.3
125	165.1	160	128.9	195	105.7
126	163.8	161	128.1	196	105.2
127	162.5	162	127.3	197	104.7
128	161.2	163	126.5	198	104.2

TABLE II.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
199	103.6	223	92.5	247	83.4
200	103.1	224	92.1	248	83.1
201	102.6	225	91.7	249	82.8
202	102.1	226	91.3	250	82.5
203	101.6	227	90.9	251	82.1
204	101.1	228	90.5	252	81.8
205	100.7	229	90.1	253	81.5
206	100.2	230	89.7	254	81.2
207	99.7	231	89.4	255	80.9
208	99.2	232	89.0	256	80.6
209	98.7	233	88.6	257	80.3
210	98.2	234	88.2	258	80.0
211	97.7	235	87.8	259	79.6
212	97.3	236	87.4	260	79.3
213	96.8	237	87.0	261	79.0
214	96.4	238	86.6	262	78.7
215	95.9	239	86.2	263	78.4
216	95.4	240	85.8	264	78.1
217	95.0	241	85.5	265	77.8
218	94.6	242	85.2	266	77.5
219	94.1	243	84.8	267	77.2
220	93.7	244	84.5	268	76.9
221	93.3	245	84.1	269	76.6
222	92.9	246	83.8	270	76.4

TABLE III. *Shewing the Distances at which different Angles are subtended by a Body* TWENTY *feet long.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
1	68755	32	2079	63	1091
2	34377	33	2084	64	1074
3	22918	34	2022	65	1058
4	17189	35	1964	66	1042
5	13751	36	1910	67	1026
6	11460	37	1858	68	1011
7	9822	38	1809	69	996
8	8594	39	1762	70	982
9	7639	40	1719	71	968
10	6876	41	1677	72	955
11	6250	42	1637	73	942
12	5730	43	1599	74	929
13	5289	44	1563	75	917
14	4910	45	1528	76	905
15	4584	46	1495	77	893
16	4297	47	1462	78	881
17	4044	48	1432	79	870
18	3820	49	1404	80	859
19	3619	50	1375	81	848
20	3438	51	1348	82	838
21	3274	52	1322	83	828
22	3125	53	1298	84	818
23	2989	54	1273	85	808
24	2865	55	1250	86	799
25	2750	56	1228	87	790
26	2644	57	1206	88	781
27	2547	58	1186	89	772
28	2455	59	1166	90	764
29	2371	60	1146	91	755
30	2292	61	1127	92	747
31	2218	62	1109	93	739

TABLE III.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
94	731	129	533	164	419
95	723	130	529	165	416
96	716	131	525	166	414
97	709	132	521	167	411
98	702	133	517	168	409
99	694	134	513	169	406
100	687	135	509	170	404
101	680	136	505	171	402
102	674	137	501	172	400
103	667	138	498	173	397
104	661	139	494	174	395
105	655	140	491	175	393
106	649	141	487	176	391
107	643	142	484	177	388
108	637	143	480	178	386
109	631	144	477	179	384
110	625	145	474	180	382
111	619	146	471	181	380
112	614	147	467	182	378
113	608	148	464	183	376
114	603	149	461	184	374
115	598	150	458	185	372
116	593	151	455	186	370
117	588	152	452	187	368
118	583	153	449	188	366
119	578	154	446	189	364
120	573	155	443	190	362
121	568	156	441	191	360
122	564	157	438	192	358
123	559	158	435	193	356
124	554	159	432	194	354
125	549	160	430	195	352
126	545	161	427	196	351
127	541	162	424	197	349
128	537	163	421	198	347

TABLE III.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
199	345	223	308	247	278
200	344	224	307	248	277
201	342	225	305	249	276
202	340	226	304	250	275
203	338	227	302	251	274
204	337	228	301	252	273
205	335	229	300	253	272
206	334	230	299	254	271
207	332	231	297	255	269
208	330	232	296	256	268
209	328	233	295	257	267
210	327	234	294	258	266
211	325	235	292	259	265
212	324	236	291	260	264
213	322	237	290	261	263
214	321	238	289	262	262
215	319	239	287	263	261
216	318	240	286	264	260
217	316	241	285	265	259
218	315	242	284	266	258
219	313	243	283	267	257
220	312	244	282	268	256
221	311	245	280	269	255
222	310	246	279	270	254

TABLE IV. *Shewing the Distances at which different Angles are subtended by a Body* THIRTY *feet long.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
1	103132	32	3224	63	1637
2	51566	33	3126	64	1612
3	34377	34	3033	65	1587
4	25783	35	2946	66	1563
5	20626	36	2864	67	1540
6	17188	37	2788	68	1517
7	14733	38	2714	69	1495
8	12896	39	2644	70	1473
9	11459	40	2578	71	1452
10	10313	41	2516	72	1431
11	9376	42	2456	73	1412
12	8594	43	2398	74	1396
13	7933	44	2344	75	1375
14	7366	45	2292	76	1357
15	6876	46	2242	77	1339
16	6448	47	2194	78	1322
17	6067	48	2148	79	1305
18	5729	49	2104	80	1289
19	5428	50	2062	81	1273
20	5157	51	2022	82	1258
21	4911	52	1984	83	1243
22	4688	53	1946	84	1228
23	4492	54	1910	85	1213
24	4297	55	1874	86	1199
25	4125	56	1842	87	1185
26	3966	57	1810	88	1172
27	3820	58	1778	89	1159
28	3683	59	1748	90	1146
29	3556	60	1719	91	1133
30	3438	61	1691	92	1121
31	3326	62	1663	93	1109

TABLE IV.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
94	1097	129	799	164	629
95	1085	130	793	165	625
96	1074	131	787	166	621
97	1063	132	781	167	617
98	1052	133	775	168	614
99	1041	134	770	169	610
100	1031	135	764	170	606
101	1021	136	758	171	602
102	1011	137	752	172	599
103	1001	138	747	173	595
104	992	139	742	174	592
105	982	140	737	175	589
106	973	141	731	176	586
107	964	142	726	177	582
108	955	143	721	178	579
109	946	144	716	179	576
110	937	145	711	180	573
111	929	146	706	181	570
112	921	147	701	182	567
113	913	148	697	183	563
114	905	149	692	184	560
115	897	150	688	185	557
116	889	151	683	186	554
117	881	152	678	187	551
118	874	153	673	188	549
119	866	154	669	189	546
120	859	155	665	190	543
121	852	156	661	191	540
122	845	157	657	192	537
123	837	158	653	193	534
124	830	159	649	194	531
125	824	160	645	195	528
126	818	161	640	196	526
127	812	162	636	197	523
128	806	163	632	198	521

TABLE IV.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
199	518	223	462	247	417
200	516	224	460	248	415
201	513	225	468	249	413
202	510	226	456	250	412
203	507	227	453	251	410
204	505	228	452	252	409
205	503	229	450	253	407
206	501	230	448	254	406
207	498	231	446	255	404
208	496	232	444	256	402
209	493	233	442	257	400
210	491	234	441	258	399
211	488	235	439	259	397
212	486	236	437	260	396
213	484	237	435	261	394
214	482	238	433	262	393
215	479	239	431	263	391
216	477	240	429	264	390
217	475	241	427	265	389
218	473	242	426	266	388
219	471	243	424	267	386
220	469	244	422	268	385
221	466	245	420	269	383
222	464	246	419	270	382

TABLE V. *Shewing the Distances at which different Angles are subtended by a Body* FORTY *feet long.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
1	137510	32	4296	63	2183
2	68755	33	4166	64	2148
3	45837	34	4044	65	2115
4	34378	35	3928	66	2083
5	27502	36	3820	67	2052
6	22918	37	3716	68	2022
7	19644	38	3618	69	1993
8	17189	39	3526	70	1964
9	15279	40	3438	71	1937
10	13751	41	3354	72	1910
11	12501	42	3274	73	1884
12	11459	43	3198	74	1858
13	10576	44	3125	75	1833
14	9822	45	3056	76	1809
15	9167	46	2990	77	1786
16	8594	47	2926	78	1763
17	8089	48	2865	79	1741
18	7640	49	2806	80	1719
19	7237	50	2750	81	1698
20	6876	51	2696	82	1677
21	6548	52	2644	83	1657
22	6250	53	2594	84	1637
23	5978	54	2546	85	1618
24	5730	55	2500	86	1599
25	5500	56	2455	87	1581
26	5288	57	2412	88	1563
27	5093	58	2370	89	1545
28	4911	59	2330	90	1528
29	4740	60	2292	91	1511
30	4584	61	2255	92	1495
31	4436	62	2218	93	1479

TABLE V.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
94	1463	129	1066	164	838
95	1448	130	1058	165	833
96	1433	131	1050	166	828
97	1418	132	1042	167	823
98	1403	133	1034	168	818
99	1389	134	1026	169	813
100	1375	135	1017	170	809
101	1361	136	1009	171	804
102	1348	137	1002	172	799
103	1335	138	996	173	794
104	1322	139	989	174	790
105	1309	140	982	175	785
106	1297	141	975	176	781
107	1285	142	968	177	776
108	1273	143	961	178	772
109	1261	144	955	179	768
110	1250	145	948	180	764
111	1239	146	942	181	760
112	1228	147	935	182	756
113	1217	148	929	183	751
114	1206	149	923	184	747
115	1195	150	917	185	743
116	1185	151	911	186	739
117	1170	152	905	187	735
118	1165	153	899	188	731
119	1155	154	893	189	727
120	1146	155	887	190	724
121	1136	156	881	191	720
122	1127	157	875	192	716
123	1118	158	870	193	712
124	1109	159	865	194	709
125	1100	160	860	195	705
126	1091	161	854	196	701
127	1082	162	849	197	697
128	1074	163	843	198	694

TABLE V.—*Continued.*

Angle in Minutes.	Feet.	Angle in Minutes.	Feet.	Angle in Minutes.	Feet.
199	691	223	616	247	556
200	688	224	614	248	554
201	684	225	611	249	552
202	680	226	608	250	550
203	677	227	605	251	547
204	674	228	603	252	545
205	670	229	600	253	543
206	667	230	598	254	541
207	664	231	595	255	539
208	661	232	592	256	537
209	657	233	589	257	535
210	654	234	587	258	533
211	651	235	585	259	531
212	648	236	583	260	529
213	645	237	580	261	527
214	642	238	578	262	525
215	639	239	575	263	523
216	636	240	572	264	521
217	633	241	570	265	519
218	631	242	568	266	517
219	628	243	565	267	515
220	625	244	563	268	513
221	622	245	561	269	511
222	619	246	559	270	509

CHAP. II.

Description of a Double-Image Telescope, and Coming-up Glass for measuring Distances.

THE theory and construction of this instrument, when employed as a micrometer for determining angles in the heavens, has already been fully considered in another part of this volume. The application of the principle to a military and naval telescope for measuring angles and distances,—the construction of the scale,—and the method of using the instrument, remain to be considered in the present Chapter.

The double-image telescope, in its common form, is represented in Plate VIII. Fig. 1. where AB, BC, CD, DE are the tubes of which it is composed. The principal object-glass is fixed at the extremity A : The moveable object-glass, which consists of two semilenses, having their centres fixed at an invariable distance, as shewn in Plate II. Fig. 6,

is screwed into the end of the tube DC, and is situated a little to the left of C; and the eye-piece is contained in the tube DE. By pushing the tubes BC, CD into the outer tube AB, the moveable semilenses near C will be brought into contact, or nearly so, with the principal object-glass at A; and distinct vision will be procured by adjusting the eye-tube DE. When this adjustment is made, two images of any object to which the instrument is directed will be distinctly seen in the field of view. If the object is so small as to occupy less than about one-third of the field, the two images will be completely separated from one another; but if it is so large as to occupy a greater portion of the field, the two images will overlap each other. When the object-glass and semilenses are thus in contact, the magnifying power of the telescope will be a *minimum*, and the angle subtended by the centres of the two images will be a *maximum*. By pulling out the tube CD, the semilenses near C will be separated from the principal object-glass;—distinct vision will again be procured by readjusting the eye-tube DE;—the magnifying power of the telescope will be increased, and the angle subtended by the centres of the images will be diminished. In like manner, by pulling out the

tube CB, and producing distinct vision by a new adjustment of the eye-piece, the magnifying power of the instrument will be still farther increased, and the angle subtended by the centres of the images diminished; but when the distance of the semilenses from the principal object-glass is equal to the focal length of the latter, the magnifying power of the telescope will be a *maximum*, and the angle subtended by the centres of the images will be a *minimum*.

By making the semilenses, therefore, move along the axis of the telescope between the object-glass and its principal focus, the centres of any two images formed by the lenses, may be made to separate or approach each other, just as if the centres of the semilenses themselves had been permitted to separate or approach each other by a motion in the direction of their common diameter.

We have already explained, in a former part of this volume, the principle of measuring angles by the contact of double images, and we have also demonstrated, that equal changes are produced upon the angle subtended by the line joining the centres of the two images, by equal motions of the semilenses along the axis of the telescope, or, in other words, that the scale which

measures the variations of the angle, is a scale of equal parts. It only remains, therefore, to point out the method of observing the angle which any portion of space subtends at the eye of the observer, —to shew how to construct the scale of the instrument,—and to describe the advantages which it possesses in measuring the distances, and the real and apparent magnitudes of objects.

If the double-image telescope is directed to any object which occupies but a small portion of the field when the semilenses and the object-glass are in contact, the two images of that object will be completely separated from each other. If the common diameter of the semilenses, or, what is the same thing, if the line which joins their centres is perpendicular to the horizon, the line which joins the centres of the two images will also be perpendicular to the horizon; or the one image will be above the other: And, in general, whatever be the inclination to the horizon of the line which joins the centres of the semilenses, the line joining the centres of the images will have a similar inclination. Hence it follows, that while the telescope is turned round its axis, the line which joins the centres of the two images will also have a corresponding rotatory motion. Now, if ABDC, Plate VIII. Fig. 7. be any object,

such as the window of a house, and if it is required to measure the angle subtended by the breadth of it AB, increase or diminish the distance between the object-glass and the semilenses, till, by turning the telescope round its axis, the upper part *a b* of the one image of the window is in contact with the upper part AB of the other image. When this happens, the index will point out upon the scale the angle subtended by AB. If the angle subtended by BD had been required, it would have been necessary to push in the semilenses, and turn the telescope about its axis till the point *d* of the one image coincided with the corresponding point B of the other image. If A, B, instead of being the extremities of any object AB, are merely separate points, such as two stars, the observation is made in precisely the same way, and the angle will be pointed out on the scale when the second image *a* of the star A is in contact with, or exactly covers the image B of the star.

Though the two images have different degrees of distinctness in different parts of the field, the observer has it in his power to make any of the images more distinct than the other, merely by changing its place in the field of view; and it will always contribute to the accuracy of the observation to bring the part of the image where

the contact is to be observed, into the centre of the field, or into that part of it where the points of contact are most distinctly seen. The observer will at first experience some difficulty in bringing the two images readily into contact, particularly when the objects are not luminous; but a little practice will remove the difficulty, and render this method of observing angles both easy and pleasant.

Before we begin to construct the scale of the instrument, which is engraven on the tubes BC, CD, Plate VIII. Fig. 1. we must first determine the extent which ought to be given to the scale, or the interval between the extreme angles, and likewise the proper value of the smallest angle, or that at which the scale should commence. The interval between the extreme angles is regulated solely by the focal length of the semilenses, and the value of the smallest angle, depends upon the distance of their centres.

If φ be the focal length of the semilenses, and b their distance from the principal focus f of the fixed object-glass, then we have (See Plate I. Fig. 5.) $GF = \frac{\varphi b}{\varphi + b}$; but when the object-glass and semilenses are in contact, b is equal to the principal focal length of the former, and GF is equal to

1

the focal length of the combined glasses. Let us suppose, therefore, that the principal focal length of the object-glass is 30 inches, and that it is required to give such a focal length to the semilenses, that the extreme angles may be to one another in the ratio of 24 to 6. As the magnifying powers are as the focal lengths, and the angles inversely as the magnifying powers, the angles will be inversely as the focal lengths; so that the focal length of the combined glasses, when they are in contact, should be to their focal length, when they are at their greatest distance, as 6 to 24; but their focal length is 30 inches in the latter case, when the semilenses are at the principal focal point of the object glass, and therefore we have

$$24 : 6 = 30 : 7.5,$$

which is the value of GF, or the focal length when the glasses are in contact. Now the formula $GF = \frac{\varphi b}{\varphi + b}$ becomes by reduction $\varphi = \frac{GF \times b}{b - GF}$, which, in the present example, gives $\varphi = \frac{7.5 \times 30}{30 - 7.5} = 10$; so that the focal length of the semilenses must be 10 inches, in order to make the extreme angles in the ratio of 6 to 24. By making the ratio of the extreme angles as $m : n$, and calling $f =$ the focal

length of the principal object-glass, we may obtain a more general formula: Thus,

$m : n = f : \mathrm{GF}$. Hence

$\mathrm{GF} = \frac{nf}{m}$; and substituting this value of GF instead of GF in the last formula, we obtain

$\varphi = \frac{bnf}{mb - nf}$, but when the semilenses are in contact with the object-glass $b = f$. Hence $\varphi = \frac{nf}{m - n}$, which in the present case is $\varphi = \frac{6 \times 30}{24 - 6}$ $= 10$ as before. The focal length of the semilenses being thus accommodated to the ratio of the extreme angles, or the interval between them, we must then fix upon a convenient value for the smallest angle, by which the distance between the centres of the semilenses must be regulated. Now, in Plate II. Fig. 4., $\mathrm{OF} = \mathrm{GF}$ in Fig. 5., and, it has already been shewn (page 32.) that AFB is the angle subtended by the objects M, N when the images of those objects are in contact at F; consequently, making a = the smallest angle, and $\delta = \mathrm{OA}$ = half of the required distance between the centres of the semilenses, we have $\mathrm{OF} : \mathrm{OA} = \mathrm{Rad.} : \mathrm{Tang.}\ \frac{1}{2} a$, and

$\mathrm{OA} = \delta = \frac{\mathrm{OF} \times \mathrm{Tan.}\ \frac{1}{2} a}{\mathrm{Rad.}}$, but

$OF = GF = \frac{nf}{m}$: Hence $\delta = \frac{\frac{nf}{m} \times \text{Tan.} \frac{1}{2} a}{\text{Rad.}}$, which, assuming $50'$ for the smallest angle, becomes $\delta =$ Log. 7.5 + Log. Tang. $25'$ — Rad. = 0.05454; which, being doubled, gives 0.10908, or nearly eleven hundredths of an inch for the distance between the centres of the semilenses. The smallest angle being taken at $50'$, the greatest angle will be found by the following analogy.

$$6 : 24 = 50' : 200', \text{ or } 3^\circ\ 20'.$$

Though we have thus determined by calculation the value of the two extreme angles of the scale, yet, as it is difficult to measure, with sufficient accuracy, the focal length of the different glasses, and the distance between the centres of the semilenses, we must have recourse to experiment for determining correctly the value of the extreme angles, as the accuracy of the whole scale depends upon the result.

In order to do this with the greatest exactness, provide two equal rectangular pieces of white pasteboard, like A, B, Plate VIII. Fig. 8., and fix each of them to a sharp pointed piece of wood or iron, in the way represented in the Figure; so that when the pins are fixed in the ground, the line AB which joins the pieces of pasteboard may nearly coincide with the upper surface of the one and the lower surface of the other. On a level

piece of ground, fix the marks A and B at such an interval from each other, that when viewed by the telescope at any distance above 200 or 300 yards, in a line perpendicular to the middle part of AB, and when the semilenses are in contact with the object glass, the lower surface of the second image of A may exactly coincide with the upper surface of B, as is represented by the dotted lines at A′ in the Figure. This contact of the images may be obtained, either by approaching to or receding from A, B; or by varying the distance between the pieces of pasteboard. When the contact is accurately observed, we may calculate the angle subtended at the object glass of the telescope, by the line AB which joins the centres of the pieces of pasteboard, by the method which has already been explained in Chap. I., Book I. By applying the correction also for the observation of focal length, computed from the formula given in the same place, we obtain a correct value of one of the extreme angles of the scale. The other extreme angle, which is formed when the semilenses are at their greatest distance from the object glass, may be determined in the very same manner. Since the scale is one of equal parts, it may be formed with sufficient accuracy without determining the angle at both extremities. Any intermediate angle will be suf-

ficiently accurate, and if the instrument is not wanted for astronomical purposes, this intermediate angle may be found with great facility, by observing when the images of the sun, taken in a horizontal line, are in contact. The sun's diameter, as found in the Nautical Almanac for the given time, will be the angle at that part of the scale. The reason of taking the horizontal diameter of the sun is to avoid the error of refraction, with which every other diameter of his disc is affected.

In the example which we have already taken, the extent of the scale, or the interval between the extreme angles, is $200' - 50' = 150'$, and the length of the scale is 30 inches, so that *two tenths* of an inch on the scale will correspond to one minute, or *one twentieth* of an inch to 15 seconds. Hence the scale may be safely divided into 600 equal parts, each of which will correspond to $15''$ of a degree; and even the fifth part of one of these divisions, or $3''$, will be perceptible to the naked eye. If greater accuracy is required with the same extent of scale, these divisions may be still farther subdivided, by means of verniers projecting from the extremities B and C of the tubes AB, BC; for when the tube ABC is completely pushed into AB, the extremity C is the index for the divisions

on CD, and when CD is completely drawn out, the extremity B is the index for the divisions on BC. As the reading off from a vernier is always troublesome, and particularly in a portable instrument, it would be preferable to diminish the extent of the scale, or the interval between the extreme angles, in order that a smaller number of seconds than 15 may be visible on the scale. Thus, if the interval between the extreme angles is 50′—20′ or 30′, then *a whole inch* on the scale will answer to a single minute, *one tenth* of an inch to 6″, *one twentieth* of an inch to 3″; hence the scale of 30 inches being divided as before into 600 parts, every three seconds will be the value of an unit on the scale, and single seconds may be easily read off by the naked eye.

The scale of the instrument being thus constructed, we should now proceed to point out its use in measuring distances; but as the application of the instrument for this purpose is made in precisely the same way as the micrometrical telescope, and as all the rules and tables which we have given for the one are equally applicable to the other, it would be altogether unnecessary to resume the subject in the present Chapter. It may be proper merely to remark, that the object, or portion of an object whose angle is measured by the method of double

images, should be chosen as luminous as possible, or when it can be obtained it should be a dark object upon a light ground, such as the top of a house with the sky behind it. The direction of the line which joins the two points that comprehend the angle may be inclined in any way to the horizon; and in measuring inaccessible distances, as represented in Plate VIII. Fig. 3., the two images of the object DE should appear separate, but as near each other as possible, when the observer is at the station B, and when the object glass and semilenses are in contact.

We have hitherto supposed that the telescope consists of four tubes AB, BC, CD, DE, Plate VIII. Fig. 1., and that by the motion of the tubes BC, CD, the semilenses approach to or recede from the object glass, while the adjustment of the tube DE produces distinct vision. We have assumed this form of the instrument, as being the general form of portable telescopes; but it is obvious that much would be gained in point of simplicity, by having only two tubes instead of three between A and D; and the facility of observation would be still farther increased, by having the tube nearest D, and likewise the eye tube DE, moveable with rack and pinion. By this means the adjustments could be made with the utmost facility.

The double image telescope possesses the same valuable property as the micrometrical instrument, in requiring no correction for the angle in consequence of the aberration of focal length, while it has the additional advantage of requiring no stand or support. When wires or steel points are made to comprehend the body, the smallest agitation removes the wires or points from the extremities of the object, and thus diminishes both the facility of making the observation, and the accuracy of the result. In the double image telescope, however, the two images always keep together, so that their contact can be observed with the utmost accuracy, not only when it is exposed to a slight tremor in the hand of the observer, but even during the motion of a vessel at sea, or during the gentle agitation of a carriage.

This valuable property renders the double image telescope of great utility, in situations where there is neither time nor opportunity for steady observations. As a naval telescope, therefore, for measuring distances in general at sea, but particularly as a *coming-up-glass* for ascertaining whether a ship is approaching to, or receding from, the observer, it is entitled to particular notice.

An ingenious instrument of this kind, invented by the celebrated Mr Ramsden, has been for

several years used in the English navy. It consists of a common telescope, having one of the eye-glasses divided into two equal segments, the centres of which are separated from each other by a micrometer screw. By this means, the centres of the two images of the object formed by the segments of the eye-glass, are separated till they come in contact. The distance between the centres of the glasses is shewn in revolutions, and parts of a revolution of the finger screw, by which they are separated. The whole revolutions are indicated by a scale on a small slip of brass fixed to the eye-tube, while each of these units is subdivided into hundredth parts, by a circular head fixed on the finger screw. Another instrument for the same purpose, invented by M. Rochon of the National Institute, and director of the naval observatory at Brest, has been used in the French navy, and found of very great utility. This instrument, founded on the double refraction of Iceland crystal, consists of two prisms of that substance, moveable along the axis of the telescope. Two images are of course formed; and the angle subtended by the line joining their centres, is measured by the distance between the prisms and the object glass. This instrument does great credit to the ingenuity of its inventor; but its utility is ex-

tremely limited by the small extent of the scale, which, without a particular construction of the prisms, cannot measure angles above 20 minutes, and also by the great difficulty of forming the prisms with sufficient accuracy. "I was desirous," says M. Rochon, "of extending the effect of the double refractions, so as to measure the diameter of the sun; and I accomplished this, which appeared to exceed the known power of the double refraction of rock crystal, which does not go beyond 20 minutes, when it is cut in the most advantageous prismatic shape. For this, I employed two equal prisms, cut in the direction most suitable to my purpose; and on placing them in opposite directions, I found that the double refraction was not perceptible; but on reversing their directions, the double refraction of each prism was nearly doubled, so that I obtained two images separated by an interval of 40 minutes. I ought not to omit, that in this new construction, there are difficulties of execution not easy to surmount."

From this it appears, that the instrument of M. Rochon will measure no farther than 20 minutes when it is made in the ordinary way, and no farther than 40 minutes when the new construction is adopted. But we have already seen, that in measuring distances, any error in the instrument,

or in the observation, produces an error in the result, which increases with the smallness of the angle, and therefore in the Iceland crystal micrometer the results must be very inaccurate. It is besides so troublesome to construct, so liable to be put out of order, and so difficult to be repaired when it is injured, that we never can expect to find it in general use.* The instrument, on the contrary, which we have described in this Chapter, may have an extent of scale above 12 times greater than that of M. Rochon's micrometer, and may

* The following quotation from the Memoir published by M. Rochon, points out the use which has been made of his instrument in the French navy.

" Conformably to these new principles, I have had two telescopes with a double refracting medium constructed under my own inspection, which General Gantheaume will employ for determining the position of his ships, and to find whether he be approaching any he may meet with at sea.

" The uses of an instrument for measuring very small angles with precision, are too well known for me to describe its advantages. The officers of the English navy are so fully aware of them, that they have used, for some years, Ramsden's eye-glass micrometer, though this answers the end but imperfectly, because it does not give the measure of the angle, and because of the bad effect of the parallax produced by the decussation of the rays that enter the eye. This defect is more sensible in Ramsden's eye-glass micrometer, than in Bouguer's heliometer. The officers who have compared my instrument with Ramsden's, of which there were several on board the Spanish ships with our Brest fleet, agree that the celebrated English artist has very imperfectly accomplished the object he proposed; and Bouguer's

measure angles even of 10 and 12 degrees. Its construction, too, is extremely simple, and it can scarcely be put out of order unless it is broken in pieces.

The method of employing it as a coming-up-glass, may be very shortly explained. When a vessel is seen at a distance, it subtends a certain angle at the eye of the observer. If its distance increases, this angle will of course be proportionally diminished; and if it approaches to the observer, the angle will be increased. Now, if this vessel be viewed through the double image telescope, and if the semilenses are separated from the object glass, till the two images either of the whole vessel or of any part of it are in contact, we obtain from the scale the angle which that part subtends. If the images continue in contact, the distance of the vessel must have remained the

heliometer would unquestionably be preferable for naval use, because it has less sensible parallax, and gives the measure of small angles, so important for determining the distances of ships from their known dimensions.

"General Gantheaume," continues M. Rochon, "has made an advantageous report of this instrument, in his account of the chace of the Swiftsure, which he captured.—This instrument is so difficult to execute, that I know only one person, citizen Narci, who is capable of giving rock crystal the prismatic form, in the proper direction for obtaining the double refractions necessary to the goodness of the micrometer."

same; but if the images have separated from each other, the vessel must be receding from the observer; and the proportion of the two distances may be found, by again bringing the images in contact, and finding the angle anew. If the first angle was 50′, and the second 40′, then the first distance must have been to the second, as 40 to 50, that is in the inverse ratio of the angles. If the images, on the contrary, had overlapped each other at the second observation, the distance between the two vessels must have been diminishing, and the degree of diminution would have been found, by bringing the images into contact, and again observing the angle. If the angle were now 60′, then the ratio of the distances must have been as 50 to 60.

Neither the length of the ship, nor any part of it lying in a horizontal direction, or in a direction inclined less than 90° to a horizontal line, should be chosen for the subtense of the angle, as the angle which it forms at the eye of the observer may vary merely from the turning of the ship, while its distance suffers no variation. The length of the masts, or any part of them, or of any vertical line in the vessel, ought always to be taken for this purpose.

CHAP. III.

Description of Luminous Image Telescopes for measuring Angles and Distances during Night.

In the fourth Chapter of Book I. we have explained the theory and general construction of this instrument. The application of the principle to a telescope for measuring distances, will form the subject of the present Chapter.

We have already seen, that the images of two luminous points formed in the focus of an object glass, may be expanded into circular images of light till they touch each, merely by pushing in the eye-piece of the telescope; and we have demonstrated, that the angle, formed by the line joining the centres of these images when in contact, varies as $\frac{\varphi f}{\varphi G}$ (See Plate III. Fig. 1.) that is, directly as the distance of the intersection of the cones of rays from the focus of the object glass, and inversely as the distance of the same point from the object glass itself. We have likewise seen, that the scale should commence at f, and stretch

towards the object glass, that is, in bringing the luminous images into contact, the eye-piece should be pushed in, so that we may perceive the sections of the cones of rays between the object glass and its principal focus, though the position of the scale upon the tube will in reality be without the focal point. The sections of the cones of light are, in general, better defined, when they are taken between the object glass and its principal focus; but, in good telescopes, the difference is not so great as to prevent the scale from being made on either side, or on both sides of the principal focus.

Aided by these principles, the construction of the instrument becomes remarkably simple. Let us suppose, that the focal length Gf of the object glass is 24 inches, and that we have determined, by actual experiment,* that the circular images are in contact, when the angle subtended by two luminous points is 4 minutes, and when φf is 6 inches; then since φG is, in this case, 18 inches, and since the angle varies as $\frac{\varphi f}{\varphi G}$, we shall obtain, by simple proportion, the angle subtended by the luminous points for any other values of φf.

* This may be done by the method described in the note at page 20, with this difference only, that two luminous points must be used, instead of the extremities B, C of the object BC, Plate II. Fig. 2.

Thus let φf be 3, and consequently φ G 21. Then

$$\frac{6}{18} : \frac{3}{21} = 4' : 1'\ 42''\ \tfrac{6}{7}$$

By making φf successively equal to 3, 6, 9, 12, 15, 18, 21, we shall obtain the results in the following Table.

Values of φf.	Values of φ G.	Calculated angles.
0	24	0° 0′ 0″
3	21	0 1 43
6	18	0 4 0
9	15	0 7 12
12	12	0 12 0
15	9	0 20 0
18	6	0 36 0
21	3	1 24 0

By this means, we obtain the value of each unit of the scale, so that we may either divide the scale into spaces proportional to the differences between the calculated angles, or, what is perhaps much better, we may make it a scale of equal parts, and take the angles from a small table calculated on purpose. The slip of brass on which the scale is engraven might be made to move in a groove, formed in one of the tubes of the telescope, in order that the index may always be set to zero, when the object is seen distinctly through the telescope; or, what is a more simple method, the scale may be engraven on the tube itself, and the index may be set to the zero of the scale at the instant of distinct vision, by pulling out or push-

ing in the object glass, which, for this purpose, should be placed in a short moveable tube. This adjustment becomes absolutely necessary, when there is any aberration of focal length arising either from the proximity of the object, or from any other cause.

It is obvious, from Fig. 1. Plate III. that the magnitude of the scale depends both upon the diameter of the object glass, and also upon its focal length. When the diameter LL of the object glass is reduced to $l\,l$, then the cones of rays LmL, LnL, are reduced to lml, lnl, which intersect each other at φ', so that we have now the space $f\varphi'$ instead of $f\varphi$, to measure the angle subtended by the luminous points M, N. If the focal length Gn of the telescope is increased to Gn', while LL remains the same, the cones of rays will cross each other at a point in the axis between φ and n, but less distant from φ than n' is from n; and hence we shall have a greater portion of the axis for measuring the angle subtended by the points. The magnitude of the scale consequently diminishes as the diameter of the object glass is increased, and increases with the focal length of the telescope. It will, therefore, depend upon the angle LnL, formed by the extreme rays of the cone.

Now let GL, Plate VIII. Fig. 9, be one half of the object glass; Gf the axis of the telescope per-

pendicular to GL; and Gn the axis of the cone of rays by which the point n of the image is formed. It is manifest, that the ray Gn, which proceeds from the object N, will pass unrefracted through the centre G of the object glass; and that the ray Gf, which proceeds from the middle point between M and N, will also pass unrefracted through the same centre G. Hence the angle nGf, which we shall call α, is equal to half the angle subtended by the luminous points M, N at the object glass G, and this angle is a constant quantity, whatever be the diameter and focal length of the object glass. In the line nG, take any points n', n'', and join $n'L$, $n''L$, crossing the axis Gf at φ' and φ''. Join also nL, cutting Gf in φ, and call the angle GnL, or half the angle of the cones, A. When Gn is the focal length of the object glass, and GnL half the angle of the cones, the interior sides of each cone will cross the axis at φ, where the images of M and N will appear in contact, and φf will be the length of scale which we have for measuring twice the angle nGf. In like manner, when Gn', Gn'', are the focal lengths of the object glass, and $Gn'L$, $Gn''L$, half the angles of the cones, the interior rays of each cone will intersect the axis at φ', φ'', where the images of M and N will appear to be in contact, and $\varphi'f'$, $\varphi''f''$, will be the length of scale for measuring the constant angle α. Now

the angle $G\varphi L = GnL + nGf = A + \alpha$, and $G\varphi$ is the tangent of $GL\varphi$, the complement of $G\varphi L$. Hence $G\varphi = \text{Cotang.}\ \overline{A+\alpha}$; and since

$$G\varphi = Gf - \varphi f, \text{ we have}$$
$$Gf - \varphi f = \text{Cotang}\ \overline{A+\alpha}, \text{ and}$$
$$\varphi f = Gf - \text{Cotang.}\ \overline{A+\alpha}.$$

from which we may obtain, when A is known, a value of α, corresponding to any length of scale φf; but it is preferable to form the scale according to the method already explained, after the value of any portion of it has been determined by actual experiment.

Hitherto we have supposed, that the angle is measured by using the intersection of the cones of light between the object glass and its principal focus; but as these cones also intersect each other beyond the focus f, the scale may sometimes be accommodated to this method of observation. In Plate VIII. Fig. 9, continue Ln till it meets the axis in Φ; then in the triangle $nG\varphi$ the exterior angle $n\varphi f$ is equal to $nGf + Gn\varphi$, or to $A+\alpha$; and in the triangle $n\varphi\Phi$ the exterior angle $Ln\varphi$, or $2A$, is equal to $n\varphi f + n\Phi f$, or to $A+\alpha + n\Phi f$, that is, $2A = A + \alpha + n\Phi f$, and $n\Phi f = 2A - A - \alpha = A - \alpha$. Hence the angle $n\varphi f$ is greater than the angle $n\Phi f$, and consequently the scale for measuring the variation in the intersection of the cones

of light beyond the focus f, is greater than the other scale; for

$$\varphi f : \Phi f = \text{Cotang. } A + a : \text{Cot. } A - a, \text{ and}$$

$$\Phi f = \frac{\varphi f \times \text{Cotang. } A - a}{\text{Cotang. } A + a}$$

A telescope, in which the aperture of the object glass is 2.25 inches, and its focal length 31.5, affords a length of scale equal to 3.33 inches, to measure a certain angle; while another telescope, in which the aperture of the object glass is 1.25, and its focal length 14.5, gives only a length of scale of 1.83 inches to measure the same angle. In the first of these instruments, one-tenth of an inch on the scale corresponds to about 30″.

From these remarks, it appears that the length of the scale depends upon the angle of the cones of rays; and that the magnitude of the scale may be increased, either by diminishing the aperture of the object glass, or by increasing its focal length. As the angle of the cones is not affected by the eye-glass of the telescope, the scale will suffer no change by any variation in the magnifying power of the instrument. This is an important advantage, as it enables us to accommodate the magnifying power to the distance and size of the luminous points, and to the intensity of their light, without requiring a new scale.

When the luminous points are truly circular,

their image in the focus of the object-glass will also have the same form, and every section of the conical frustum comprehended between the object-glass and the image, will also be circular when it is perpendicular to the axis of the telescope. This, however, is strictly true, only when the image is formed in the axis of the telescope; for in all other cases, when the image is formed towards *n* or *m*, the axis of the cone is not perpendicular to its base. But even in these cases, the deviation is too trifling to be noticed, as it does not affect the mensuration of the angle subtended by the centres of the images.

If the luminous points are not circular, but have any other form, as N in Plate VIII. Fig. 10; then if LL is the object-glass, an image similar to N will be formed at *n*. In this case, the rays by which the image *n* is formed, have not a conical shape as before, but are refracted into a solid, bounded on one side by a circle LL, and on the other by the irregular figure *n*. Every section of this solid, however, approaches to a circular form, in proportion to its proximity to LL; and as the image *n* is extremely small compared with LL, any section of it *ab* will not deviate perceptibly from a circle. Hence we perceive the reason why the most irregular pieces of light, whether of an oblong, a triangular, or a quadrilateral form, always

swell into circular images when viewed through telescopes in which the eye-glass is moved from the place of distinct vision either towards or from the object-glass of the telescope. In the luminous image telescope, therefore, it is of the greatest consequence that the apertures be exactly circular; and when the form of the luminous points can be altered by the observer, they should likewise be made as round as possible.

As the accuracy of observation must in a great measure depend on the distinctness of the line by which the circular images are bounded, it is of the utmost importance that an excellent achromatic object-glass should be selected for the instrument. The objects being always luminous, there is no occasion for much light, and a great magnifying power being equally unnecessary, the diameter of the object-glass may be made much smaller than in common telescopes. By this means we, in a great measure, remove that indistinctness in the outline of the circular images, which arises either from spherical aberration, or from the uncorrected aberration of refrangibility. The diminution of the aperture too, as we have already seen, has the advantage of increasing the length of the scale.

When the instrument is constructed according to these principles, its application to the measure-

ment of distances is precisely the same as that of the micrometrical telescope and the double image telescope; and the Tables, which we have given in the first Chapter of this Book, may be advantageously employed along with the luminous image telescope.

The use of this instrument, however, is in some respects limited, from the necessity of having the angle always subtended by luminous points. But there are numerous cases, and these too, of great importance, where it may be successfully employed. The distance of a vessel from the coast, or the rate at which it approaches to or recedes from the shore, may frequently be determined by taking the angle subtended by accidental or by fixed lights on the land.

On the coast of Great Britain there are various lighthouses, which have two lights placed at a considerable distance from each other. The principal of these are contained in the following Table, for which I am indebted to Robert Stevenson, Esq. civil engineer. In the first column are given the names and situation of the light houses; the second column contains their supposed distance, which is in general pretty correct; and the third the bearing of the one light with respect to the other.

Names and situations of the Light-houses.	Supposed distances of the Lights.	Bearings of the Lights relatively to each other.
Pentland Skerries, Orkney,	60 feet	N. W. & S. E.
Tay Lights, Forfarshire,	$\frac{1}{4}$ of a mile	N. W. $\frac{5}{4}$ N.
Fern Isles, Northumberland, ...	$1\frac{3}{4}$ miles	E. N. E. & W. S. W.
Tyne Lights, Northumberland, ...	$\frac{1}{8}$ of a mile	N. W.
Spurn Lights, Yorkshire,	$\frac{1}{6}$ of a mile	N. W. by W. $\frac{1}{4}$ W.
Hasbrough Lights, Norfolk, ...	$\frac{1}{3}$ of a mile	N. W. $\frac{1}{4}$ W.
Winterton Lights, Norfolk, ...	$\frac{1}{4}$ of a mile	W. S. W. $\frac{1}{4}$ W.
Leostoff Lights, Suffolk,	$\frac{1}{2}$ mile	S. $\frac{3}{4}$ E.
Orfordness Lights, Suffolk,	$\frac{3}{4}$ of a mile	N. E. by E.
South Foreland Lights, Kent, ...	$\frac{1}{6}$ of a mile	E. & W.
Portland Lights, Portland Isle, ...	$\frac{1}{2}$ mile	N. E. $\frac{1}{2}$ E.
Lizard Lights, Cornwall,	$\frac{1}{8}$ of a mile	W. N. W.
St Anne's Lights, Pembrokeshire, ...	200 feet	S. S. E.
Lake Lights, Cheshire, .	$\frac{1}{4}$ mile	S. S. W.
Bidstonehill Lights, Cheshire, ...	3 miles	S. E. by E.

Dudgeon
Newarp
Nore
Goodwin
Owers

} Floating Lights, from 25 to 30 feet.

One of the great objects of these double lights, is to enable the mariner to distinguish them from others for which they might be mistaken; an effect which is produced in some cases by revolving and coloured lights. We conceive, however, that a more extensive system of double lights might be introduced with great advantage, and that these might be distinguished from one another by the different angles which a line joining the lights forms with the horizon.

The principal object of the luminous image telescope, is to make surveys; or, in general, to measure angles and distances during the night, when natural objects cannot be seen, either through a telescope, or with the naked eye. For this purpose, artificial lights, or luminous points must be used; and when the distances to be measured are short, these might be fixed upon a horizontal arm, supported by a vertical pole; but when the distances are great, the artificial lights should be placed at the extremities of a chain, 20, 30, 40, or 50 feet long. The chain being carried by an assistant, and stretched in a direction perpendicular to a line joining its centre and the eye of the observer, the telescope is directed to the luminous points, which, by the method already described, are expanded into circular images, till they come

into contact. The angle which these points subtend at the observer's eye being thus ascertained, their distance may be immediately determined by means of the Tables in Chapter I.

For military purposes, this method of measuring distances during night, may frequently be of the greatest advantage. If the lights are muffled up on the side next the enemy, the whole operations might be performed with the most profound secrecy, and the survey of a piece of ground might be made with a degree of deliberation which could not be exercised during the day-time, when the engineer is within reach of the enemy's guns. The luminous image telescope requires no stand like the micrometrical telescope, as the circular images will remain in contact during any agitation of the instrument; and in using it, the observer is not troubled with double images, or with double adjustments, as in the divided object-glass micrometer. This instrument possesses also another advantage, as it can measure the angle subtended by two luminous points, even when these points are too distant to be comprehended in the field of the telescope. In the micrometrical telescope, the two points must always be seen in the field of view; but it is sufficient in the present instrument, that the observer perceives the

contact of the circular images, without seeing their centres.

When artificial lights are used, it is obviously in our power to diminish the aberration of refrangibility, and thus to render the outline of the circular images more perfect, by employing homogeneous lights, such as red or orange lights, in which there are few or none of the most refrangible rays.

This instrument might also be employed to measure the relative distances of any object photometrically, and when there is only one luminous point. When light is projected from any luminous body, its intensity varies in the inverse ratio of the square of the distance. By observing, therefore, the distance of the index from the principal focus, when the circular image is expanded to such a degree that it ceases to become visible, it is easy to calculate the degree of attenuation which the light has experienced. The same observation being made at a different distance from the luminous object, we obtain the ratio of the squares of the distances, and consequently the ratio of the distances themselves. This will be better understood when we have described the photometer in a subsequent part of the work.

In looking at the circular images formed by

the expansion of luminous points, a variety of curious optical phenomena arise from opening and shutting the eye, and from holding it in different positions. The explanation of these facts, however, will be more properly introduced when we come to describe the construction of a general Photometer.

CHAP. IV.

Description of Instruments for measuring Inaccessible Distances at one Station.

The instruments described in the preceding chapters, and indeed all the trigonometrical instruments with which we are acquainted, are capable of measuring inaccessible distances, only from observations taken at the extremities of a real base, or of a base which is included in the instrument itself. The impossibility, which sometimes exists, of finding ground sufficiently level for the mensuration of a straight line, and the trouble and difficulty which attend the operation even when the ground is favourable, render it extremely desirable to have some instrument by which inaccessible distances can be determined from a single station. The method of measuring distances, by observing the variation in the focal length of a telescope, is totally inadequate for this purpose; and even if it were not affected with errors which do not admit of correction, it could

be applied only to distances that are extremely small.

The principle upon which one of the following instruments is constructed has been long understood, but, so far as I know, its application to the measurement of angles and distances has never yet been attempted. If two circles AB, CD, of different radii, Plate IX. Fig. 1. are fixed upon an axis EQ passing through their centres, and if these circles are made to roll upon a smooth surface, they will describe arches of a great circle whose centre is at the point O, formed by the intersection of the lines AC, BD, which join the extremities of any two parallel diameters of these circles. If one of the circles CD is shifted into the position *cd*, the circumferences of AB and CD will describe portions of a larger circle, as the intersection of the lines A*c*, B*d* will take place at a point considerably beyond O.

In order to find an expression for the radius of the circles described by this instrument, draw C*n* parallel to EF, then since the triangles A*n*C, CFO are similar, we have

$\mathrm{CO} : \mathrm{CF} = \mathrm{AC} : \mathrm{A}n$, and

$\mathrm{AO} : \mathrm{AE} = \mathrm{AC} : \mathrm{A}n$, from which we obtain

$$\mathrm{CO} = \frac{\mathrm{CF} \times \mathrm{AC}}{\mathrm{A}n}, \text{ and}$$

$$\mathrm{AO} = \frac{\mathrm{AE} \times \mathrm{AC}}{\mathrm{A}n}.$$

Hence it appears, that the radius of curvature varies directly as the distance between the circumferences of the circles, and inversely as the difference of their radii.

Let us now suppose that the circles AB, CD, instead of being placed upon an axis EQ, are fixed upon the tube of a telescope, and that the axis of the telescope coincides with the line joining the centres of the circles. If an instrument thus constructed, is made to roll upon a smooth surface, its axis will uniformly point to the centre of the circles described by AB, CD, and if the radius AO is accurately determined, the space described by any point in the circumference of AB will be a measure of the angular motion of the axis of the telescope. In order to measure with accuracy the space described by any point in the circumference of AB, the instrument is fitted up as represented in Plate IX. Fig. 2, where AB is one of the rolling circles fitted upon one extremity of the telescope, and E the eye-glass. A level L is fixed upon the arm LV, which moves round E as a centre, by means of the endless screw SS working in the teeth of a wheel attached to the arm LV. The vernier V is placed at the other extremity of the moveable arm, and subdivides, into 10 parts, each of the divisions on the graduated circumference AB, so that the thousandth part of

the whole circumference may be distinctly read off. In order to explain the method of using the instrument when thus constructed, let us suppose that the axis of the telescope is marked by the extremity of a sharp steel point, projecting from the circumference to the centre of the field of view, and that the instrument is placed upon a smooth and level surface, upon which it can roll without any slide. Let ABCD, Plate IX. Fig. 3, be the telescope thus fitted up; and having the index of the vernier pointing to the zero of the circular scale, direct it to any object MN, and adjust it to such a position by the hand, that the steel point exactly covers any prominent part N of the object when the axis of the level is horizontal. Roll the instrument along the plane surface, till the steel point comes into coincidence with any other prominent part M of the object. The instrument will now be in the position *a b c d*, and A *a* will be the arch which it has described. The number of revolutions performed by the circle AB are easily counted; and if the level is not horizontal when the steel point coincides with M, the instrument must have described part of another revolution. In order to find this portion, turn the screw SS till the level indicates a horizontal position, and the index of the vernier will then point out on the scale the hundredths or thousandths

of a revolution by which the space A *a* exceeds the number of complete revolutions. The exact diameter of AB being known, and the radius A *o*, the space A *a* will be determined, and consequently the angle at O, or the angle subtended by the points MN at the centre of the arch A *a*. The scale upon AB might easily be divided into minutes, a whole revolution representing a certain number of degrees, so that the angle at O could be read off with the utmost facility. By means of this instrument, therefore, we are enabled to measure the angle subtended by any object at a point *before* the place of the observer.

Let the graduated rolling circle AB be now removed to CD, the object end of the telescope, as is represented in Plate IX. Fig. 4, while the other circle CD takes the place of AB. If the telescope, when thus altered, is directed to the object M′N′ as before, and is made to roll upon a plane surface, the arch A′*a*′, which it describes, while the steel point moves from a state of coincidence with M′ to a state of coincidence with N′, will obviously be measured in the way which has already been explained; but the angle to which it corresponds will now be the angle formed at O′; a point as far *behind* the observer, as the point *a* was formerly *before* him. We have therefore the angle subtended by the ob-

ject M′N′ at the points O, O′, and also the intermediate distance OO′=2R, R being the radius of the arch A*a* or A′*a*′; consequently the distances PO, PF, and PO′ are known. Calling the angle M′ON′=*m*, the angle M′O′N′=*n*, and R the radius of the arch A*a* or A′*a*′, we shall obtain

$$PO=\frac{2Rn}{m-n}$$

$$PF=\frac{2Rn}{m-n}+R$$

$$PO'=\frac{2Rm}{m-n}.$$

If the shifting of the rolling circles from the one extremity of the telescope to the other should be regarded as a source either of error or inconvenience, the second observation might be taken by another instrument, in which the largest rolling circle AB is fixed upon the object end of the telescope, the radius of the instrument being either the same as that of the former, or different from it. If the radius of the first telescope is different from that of the second, we may call it R′, and then we shall obtain

$$PO=\frac{n\times\overline{R+R'}}{m-n}$$

$$PF=\frac{n\times\overline{R+R'}}{m-n}+R$$

$$PO'=\frac{m\times\overline{R+R'}}{m-n}$$

In an instrument of this kind which I have constructed, the diameter of the largest roller is $\frac{82.57}{30.00}$ of an inch, the diameter of the smallest roller $\frac{78.79}{30.00}$, the difference of their radii $\frac{1.89}{30.00}$, and the distance between the rollers $\frac{737.50}{30.00}$. The radius is therefore 44 feet $9\frac{37}{100}$ inches, and the length of the base OO′ 89 feet $6\frac{3}{4}$ inches. In several experiments which I made with this instrument, the rolling circles described their respective arches without the smallest slide, and the successive measurements of these arches differed from each other by quantities too minute to be noticed.

The principle upon which this instrument is constructed, may be applied in another form, where the rolling of the telescope is rendered unnecessary. Thus, in Plate IX. Fig. 5, let AB, CD be two equal and concentric arches, firmly bound together, and divided into degrees and minutes, and let the telescope EF, carrying a vernier scale at both its extremies E and F, be so placed between AB and CD, that by means of a screw its axis may be brought into a state of coincidence with any radial line in the arches. This coincidence will obviously be indicated by the two verniers pointing out the same degree and minute

upon each arch. In measuring, therefore, with this instrument, the angle MON, Plate IX. Fig. 3, from the centre O of the two arches, adjust the two verniers to the zero of each arch, and then move the whole instrument till the steel point projecting from the diaphragm of the telescope exactly covers the point M of the object. The instrument being now kept steadily in this position, remove the telescope towards the other extremities of the two arches, till the steel point coincides with the other point N of the object, at the same time that the two verniers mark the same degree and minute in each scale: The degree and minute now indicated by the verniers will be the angle required. In order to measure the angle subtended at a point behind the observer, we have only to invert the instrument, by placing the arch AB nearest the object, and also to invert the telescope, so that the vernier at L, the object end of the tube, may be placed upon the other arch AB.

The same principle is capable of another application, which is not without some peculiar advantages. Two telescopes may be absolutely fixed at an invariable distance, and their axes may be changed by means of two micrometers, in such a manner that the intersections of the axes may take place either at a point before or behind

the place of the observer. If the axes of the fixed telescopes A, B, Plate IX. Fig. 6. be in the direction GO′, HO′, they will obviously intersect each other at a point O′ behind the observer, and the radius BO′, as well as the angle AO′B′, will be determined by the micrometer. If the axes of the telescopes are now moved by means of the micrometer screw, into the directions AO, BO, they will now intersect each other at a point O before the observer, and the radius AO, together with the angle AOB, will be determined as before by means of the micrometer.

The principle of all these instruments may be applied, in different forms, to a variety of useful purposes; but there is one application of it in particular, which promises to be of some practical importance. If the two telescopes are fixed at an invariable angle, so that their axes always point to the same part of an object, they will obviously form a portable base, and will thus enable us to set off a distance without the aid of any other instrument, and under circumstances, when a base could not be measured by a chain or by glass rods. The instrument will give the distance through the air, unaffected with the superficial inequalities of the ground, and in a line parallel to the horizon; and whatever alterations it may undergo by a change of temperature, a correction founded on

experiment can be easily applied. It is obvious that any error in the observation will produce a very considerable error in the base, and that the latter error will be to the former, as radius is to the sine of half the angle formed by the telescopes. The error of observation, however, must always be very small when the telescopes are good, and cannot be supposed to exceed the hundredth of an inch, when the point to which the axes of the telescopes converge is not very remote. At all events, we have it in our power to diminish, as much as we please, the error which affects the base, by increasing the distance between the telescopes, and by enlarging their magnifying power. If the length of the base is about 1000 feet, the distance between the telescopes about three or four feet, and their magnifying power about 100, we conceive that the error in the base would scarcely amount to a single inch.

CHAP. V.

Description of an Optical Instrument for measuring inaccessible Distances at one Station.

In the preceding Chapter, we had occasion to allude to a method of estimating distances, from the variation in the focal length of a telescope. When a telescope is directed to an object very remote, such as one of the heavenly bodies, the image of such an object will be formed in the principal focus of the object glass. If this object is supposed to approach gradually to the telescope, its image will be formed at a greater distance from the object glass than its principal focus, and every variation in the distance of the object will be accompanied with a corresponding variation in the distance of the image. The ratio of these variations being known from the principles of optics, the distance of the object may be easily determined by the distance of its image from the principal focus of the

object glass. If D be the distance of the object from the telescope, and F the principal focal length of the object glass, the aberration or increase of focal length will be $\frac{F^2}{D-F}$. Let F be equal to 24 inches, and let D be successively 100, 500, 1000, 2000, 3000, 4000 feet, &c. we shall obtain the following values of the aberration of focal length :

Distance of the object in Feet.	Variation of focal length in Inches and decimals.
100 - - - - - -	5.878
500 - - - - - -	1.157
1000 - - - - - -	0.578
2000 - - - - - -	0.289
4000 - - - - - -	0.144
8000 - - - - - -	0.072
16000 - - - - - -	0.036

From these results, it appears that, at the short distance of 100 and 500 feet, the variation of focal length is sufficiently rapid, and might be considered as a rough measure of these distances; but when the distances increase, the variations are so extremely small, that we have only a variation of $\frac{7}{100}$ of an inch, as a scale for measuring every distance between 4000 and 8000 feet. Hence we may fairly conclude, that the variation of focal

length can only be employed to measure distances that are extremely small.*

But without considering the extreme minuteness of the scale, this method of measuring distances is liable to great uncertainty, from variations of temperature, and from changes in the state of the observer's eye. The effects of these variations might indeed be removed by using a moveable scale, and adjusting the zero to the principal focal point of the telescope at the time of observation; as determined by directing it to a very remote object: But this method of correction would render too complicated, an instrument which derives from simplicity of construction any merit that it may possess.

The magnitude of the scale can be increased to any extent, by keeping the object-glass and eye-glass at an invariable distance, and producing the adjustment to distinct vision, by the motion of a second object-glass of considerable focal length; or the same thing might be effected in the Grego-

* For the purpose of measuring small distances, the aberration of focal length might be employed with great advantage. A small telescope, fitted up with a tube about double the focal length of the object glass, would measure, with great accuracy, the dimensions of buildings, the breadth of rivers, and other short distances, which it is often both interesting and useful to determine.

rian reflecting telescope, by keeping the two mirrors at a constant distance, and producing distinct vision by the motion of the eye-piece, as in the reflecting micrometer, described in Book I. Chap. II. In the micrometrical telescope, too, a similar enlargement of the scale may be obtained, if we procure distinct vision by a motion of the tube which contains the moveable object-glass. The eyepiece will of course move along with it, but the aberration of focal length will be considerably expanded. In all these contrivances, however, and indeed in every contrivance which can be devised for this purpose, the imperfections of the instrument are magnified along with the scale, so that no advantage whatever can be derived from its enlargement. If there is any uncertainty respecting the focal point within the space of the hundredth of an inch in the scale of the first object-glass, and if this scale is magnified 100 times by any optical contrivance, the uncertainty must now be extended over a whole inch, and consequently the instrument must in reality be deteriorated by every attempt to increase the scale.

Knowing this to be the case, both from theory and experiment, we were not a little surprised at seeing the proposal made by the late Mr Patrick Wilson, formerly professor of astronomy at Glas-

gow, of determining the different distances of a star, by the variations in the focal length of a long telescope.* We have no opportunity of knowing what particular method was employed by Mr Wilson, for magnifying the aberrations of focal length, but it is highly probable that they were similar to those which we have already mentioned; or, at all events, they must have been so contrived, that the focal point of one object-glass was made the radiant point to another. Whatever the contrivance was, we have no hesitation in saying that it was quite useless; and if a telescope could be constructed of the same length as the diameter of the earth's annual orbit, we conceive it quite demonstrable, that the variations of its focal length, however magnified, could never be of any service in ascertaining the different distances of a fixed star.

This property of measuring distances is by no means peculiar to the telescope, as has been commonly imagined. Considering the principle ge-

* This proposal was recorded in the Transactions of the Royal Society of Edinburgh, Vol. i. Hist. p. 30, by the learned Professor Robison. "If a long achromatic telescope be directed to a fixed star, towards which the earth is moving, the focal distance of the telescope will be lengthened. The augmentation will indeed be very small; but Mr Wilson has fallen upon very ingenious methods of increasing it, so as to make it become sensible."

nerally, it is nothing more than to measure any variation in the tangent of the angle of incidence, from the corresponding variations in the tangent of the angle of refraction, or of the angle of reflection, if a reflecting telescope is employed. When this principle is embodied in the telescope, it is by no means applied in its best form. The limit which is necessarily put to the diameter of the object glass, and all the imperfections of the instrument, whether they arise from the nature of the glass, or from the defects of its figure, concur in affecting the measure of every change in the tangent of the angle of refraction. We propose, therefore, to extend the application of the principle, by employing two plane mirrors, placed at a considerable distance, and converging to a point behind the observer. The variations in the tangent of the angle of reflection may then be accurately determined, by means of a small telescope moveable round a centre, and capable of being carried along the axis of the instrument, or the line which is equidistant from the two mirrors. The same effect will be obtained, by employing two prisms, placed at a distance from each other, in such a manner, that the refracted rays, proceeding from a remote object, may meet in the line which is equidistant from the two prisms.

As instruments of this kind can never be very portable, and are therefore not likely to be constructed for general use, we have thought it sufficient merely to notice the principle of their construction.

BOOK IV.

ON

OPTICAL INSTRUMENTS

FOR DIFFERENT PURPOSES,

IN WHICH THE

RAYS ARE TRANSMITTED THROUGH FLUIDS.

CHAP. I.

Description of Instruments for viewing Objects under Water.

THE subject of this Chapter was suggested to me some time ago, by a notice in one of the Philosophical Journals, that the Academy of Sciences at Copenhagen had offered the mathematical prize to the inventor of a hydraulic tube, by means of which objects might be distinctly seen at the bottom of the sea. The great practical utility of an instrument of this kind, and the advantages which may arise from applying the principle upon which

it is constructed to other optical instruments, induced me to give it a degree of attention to which it might not otherwise have been entitled.

If the surface of the sea is ruffled, or in a state of agitation, it is obviously impossible to see the objects below by any instrument which is not immersed in the water. Even when the surface is perfectly smooth, and the objects below sufficiently illuminated, they can be perceived distinctly, only when the visual line forms a very considerable angle with the surface. It is possible, indeed, to have a contrivance which will counteract the oblique position of the surface with regard to the axis of vision; but as the correction must always vary with the angle of obliquity, and must be adapted to that angle, it cannot be applied in practice till the inclination of the visual line to the surface of the water has been previously ascertained by a geometrical operation. No instrument, therefore, for seeing objects under water, can be of any service, in which the rays proceeding from the objects below must be transmitted through the surface of the fluid. Hence it becomes a necessary part of the construction, that the instrument shall be partly immersed in the water, and that it shall either float by itself, or be attached to some floating body.

Admitting this, therefore, as a principle indispensibly requisite in an instrument of general application, we shall proceed to describe the different contrivances which seem to fulfil the conditions of the problem.

Let SS, Plate X. Fig. 1. be the surface of the sea, ruffled by a breeze, or in such a state of agitation as to prevent the objects below from being seen by an eye out of the water. Into the parallelopiped MN, made of wood or hollow copper, let the tube ABCD be inserted in such a manner that it has a motion round the pivot P, in a vertical plane, and that one half of it is plunged into the water. Near the lower extremity CD of the tube, fix a piece of thin and well-polished plate-glass *ef*, at right angles to the axis of the tube, and cemented to it, so as to resist the admission of the water. When this apparatus is plunged into the sea, the floating parallelopiped MN will keep the tube ABCD in a vertical plane, and by moving it round the pivot P, it may be directed to any object at the bottom. The water contiguous to *ef* being pressed close to the plate of glass, by the weight of the superincumbent column, the rays from the object will proceed, almost without suffering any change, to the observer at O, and will give as distinct a view of

it as if the surface of the water were perfectly smooth, and the eye placed directly above the object. By this means, the irregularities arising from a broken or agitated surface, or from the oblique position of the surface when it is perfectly smooth, are completely removed.

If the depth is so great, or the objects so small, that the assistance of the telescope is necessary to perceive them distinctly, an instrument of this kind may be placed in the tube ABCD, and used in the common way. As this form, however, might rather be awkward and inconvenient, it would be advantageous to apply the telescope in the way represented in Fig. 2, where MN is the floating parallelopiped, and AB the telescope, moving round the pivot P, and having its tubes nearly twice as long as the focal length of the object-glass. The object-glass AD being fixed in such a manner as to prevent any water from insinuating itself into the tube, and being sunk below the surface of the sea, will be in the same state as the plate of glass in the preceding instrument, and will transmit a perfect image of the objects below to the eye at B. As the rays proceeding from the object pass from water to glass, they will obviously suffer less refraction at the confines of the glass and water, than if air had been the medium adjacent to th

object-glass. Hence the focal length of the telescope will be considerably increased, and its achromatism in some degree impaired. The effect which this will produce upon the goodness of the telescope, however, is too trifling to be noticed, when we consider the coarse purposes to which it is applied. But if it is thought of sufficient importance, it is easy to adapt the curvature of the lenses to this particular case of refraction, and to accommodate it to the formulæ of D'Alembert and Boscovich. This may be done very easily, by dividing the radius of curvature a of the outer surface of the object-glass by 2.65, so that the rays in passing from water into the surface, whose radius is $\frac{a}{2.65}$, will suffer the same changes by refraction, as if they had been transmitted from air into a surface whose radius was a.

Thus in Plate X. Fig. 3. let BAD be a convex surface of crown-glass, whose centre is C, and let parallel rays of light incident at B be refracted to F; then, by the principles of Optics,

$$AF : FC = m : n, \text{ and}$$
$$AC : AF = m - n : m$$

m representing the sine of the angle of incidence, and n the sine of the angle of refraction. When the refraction is made from air to glass, we have

$$AF : FC = 1.53 : 1, \text{ and}$$
$$AC : AF = 0.53 : 1.53.$$

But when the refraction is made from water to glass, we have

$$A'F' : F'C' = 1.15 : 1,*$$
$$A'C' : A'F' = 0.15 : 1.15.$$

Hence, when the radius of curvature AC, or $a = 1$, we obtain

$$a : AF = 1 : 2.887 \text{ from air to glass,}$$
$$a' : A'F' = 1 : 7.667 \text{ from water to glass.}$$

When $AF = A'F'$, we have

$$a : a' = 7.667 : 2.887,$$
$$\text{and } a' = \frac{a \times 2.887}{7.667} = \frac{a}{2.65}.$$

That is, in order that a ray of light incident upon a spherical surface at B may be refracted to the same point F, when the refraction is made from air to glass, and from water to glass, the radius of curvature BC must in the one case be a, and in the other $\frac{a}{2.65}$.

It is obvious, that the increase in the focal

* If we suppose the index of refraction from air to crown-glass to be 1.53, and that from air to water to be 1.336, then the index of refraction from water to crown-glass will be 1.15, or the sines of the angles of incidence and refraction will be as 1.15 to 1.

length of the object-glass of the telescope, will depend upon the curvature of that surface of the compound lens which is in contact with the water. If its radius of curvature were infinite, that is, if the surface were plane, the incident rays would suffer no refraction, whether they were transmitted through air or water, and consequently the focal length of the compound lens would suffer no change by immersion in water, as the refraction at all the other surfaces by which the rays are converged to the focus, is the same as when the instrument is used in the open air.

Now, if F is the focal length of the compound object-glass, and a, b the radii of the two surfaces of a convex lens of crown-glass, whose index of refraction is 1.53, we have, by the principles of Optics,

$$F = \frac{1.887\, a\, b}{a + b}, \text{ and consequently,}$$

$$a = \frac{F\, b}{1.887\, b - F}$$

$$b = \frac{F\, a}{1.887\, a - F}.$$

But in every compound object-glass, whether double or triple, the focal length F is known, and the radius a of the surface next the object may be easily found. Hence we may determine the radius b of an equivalent surface, which will pro-

duce the same ultimate refraction of the rays, that is produced by all the other surfaces of the compound lens. Thus, if $F = 12$, and $a = 8$, and consequently $b = 24$, then a convex lens, the radii of whose surfaces are 8 and 24, will have nearly the same focal length with any compound object-glass having its focal length 12, and the radius of its outer surface 8.

We have already seen that the focal length of a convex surface of crown-glass when immersed in water, and when its radius is $\frac{a}{2.65}$, will be the same as when its radius is a, and when the refraction is made from air into crown-glass; and, consequently, the focal length of any convex lens, or compound object-glass, when the surface a is immersed in water, will be

$$F = \frac{2.65 \times 1.887 \times a\,b}{2.65\,a + b} = \frac{5\,a\,b}{2.65\,a + b},$$

b representing the radius of the other surface, or of a surface equivalent to all the remaining surfaces of the compound object-glass. When the lens is equally convex, we shall have

$$F = 1.37 \times a.$$

We are therefore enabled, by the preceding formulæ, to find the length of the tube necessary for any object-glass when a and F are known, upon the supposition that the object is infinitely distant.

As the distance of the object, however, must necessarily be very small, the length of tube L which the object-glass will, on this account, require, may be found from the following formula:

$$L = \frac{FD}{D-F},$$

where D is the shortest distance, or depth of water in which the instrument is to be used, and F the focal length of the object-glass, when immersed in the fluid.

Since it is of great consequence to have the tube as short as possible, the radius of curvature of the outer surface *a*, should be made as great as is consistent with the achromatism of the compound lens; for the increase in the focal length of the object-glass, when immersed in water, varies as the curvature of that surface.

When the objects at the bottom are sufficiently illuminated by the light of the sky transmitted through the water, the instruments now described will enable us to perceive them with almost the same distinctness, as if they had been situated at an equal distance from the observer in the open air. But if the inequalities of the surface should obstruct the transmission of the incident rays, or if they should be in a great measure absorbed in the mass of water through which they must tra-

vel, it will be necessary to render the subjacent objects visible by artificial illumination.* For this purpose, a tube GHEF, Fig. 4, must be attached to the tube ABCD, Fig. 1, or to the tele-

* When the sky is clear, and the surface of the water unruffled, objects may in general be seen distinctly at the depth of 50 or 60 feet, and sometimes at a much greater depth, without the aid of artificial illumination. "By the glass window," says Dr Halley, "so much light was transmitted, that, when the sea was clear, and especially when the sun shone, I could see perfectly well to write or read, much more to fasten or lay hold of any thing under us to be taken. And by the return of the air-barrels, I often sent up orders, written with an iron pen on small plates of lead, directing how to move us from place to place, as occasion required. At other times, when the water was troubled and thick, it would be dark as night below; but in such case, I have been able to keep a candle burning in the bell as long as I pleased, notwithstanding the great expence of air requisite to maintain flame." *Phil. Trans.* 1716, vol. xxix. p. 498. In another place, the same ingenious author observes: "As to seeing under water, as long as the water is not turbid, things are seen sufficiently distinct; but a small degree of thickness makes perfect night at no great depth of water." *Phil. Trans.* 1721, vol. xxxi. p. 179. In some of the lakes of North America, the transparency of the water is so great, that objects can be perceived at a very unusual depth. "The waters of Lake Superior," says Mr Heriot, "are more pure and pellucid than those of any other lake upon this globe, and the fish, as well as the rocks, can be distinctly seen at a depth incredible to persons who have never visited these regions. The density of the medium on which the vessel moves, appears scarcely to exceed that of the atmosphere; and the traveller becomes impressed with awe at the novelty of his situation." Heriot's *Travels through the Canadas.*

scope AB, Fig. 2. At the lower extremity of the second tube is placed a piece of plate-glass EF, inclined a little to the axis of the tube; and at the upper extremity is a parabolic mirror GH, with a lamp I placed in its focus, and a conical top K, for letting out the smoke. The rays emitted by the lamp are reflected from the mirror GH in parallel lines, and, falling with a small degree of obliquity upon the glass EF, are refracted from their original direction, so as to illuminate the objects below, to which the tube is pointed. In order to direct this refracted light to the object, whatever be its distance from the observer, it will be necessary to have a screw communicating a gentle inclination to the tube GF. A small degree of divergency may be advantageously given to the refracted rays, by making the interior surface of the glass EF concave. Instead of using the light of the lamp, the light of the sky may be admitted at the open end of the tube GH. When the sea is thick and turbid, the light of the sky is obstructed by the solid particles which are dispersed through the fluid mass, and hence it is impossible, by any contrivance, to illuminate the objects at the bottom. This instrument might be conveniently used along with a diving-bell; and in place of being attached, as is represented in the figure,

to a floating piece of wood, it might with greater advantage be fixed on the gunwale of a boat.

As it may sometimes be necessary to examine objects situated below a projecting rock, or beneath the boat or vessel in which the observer is placed, the form of the instrument must be accommodated to the peculiarities of this mode of observation. With this view, the plates of glass EF, *ef*, Plate X. Fig. 5, must be fixed in the sides of the tubes, and the two plane mirrors MN, *mn*, placed at an angle of 45° to the axis. The objects below will thus be illuminated, and seen in the same manner as before, particularly when they are obliquely placed with regard to the observer. A loss of light, indeed, will be sustained, in consequence of the reflection from the plane mirrors; but this unavoidable evil is compensated by the advantages peculiar to the form of the instrument.

When the instrument represented in Plate X. Fig. 2. is employed for viewing objects beneath a projecting rock, the plane mirror may be either placed without the telescope, at a little distance from the object-glass, so as to form different angles with the axis of the instrument; or the eyepiece of the telescope may be bent into a rectangular form, and the mirror placed at the angular

point, so that it may make an angle of 45° with the axis of each tube. This form of the instrument is perhaps the most convenient, even for general purposes.

It would lead us into too wide a field to detail the various purposes to which instruments of this kind might be applied; but some of these are of such peculiar importance, as to require particular notice. As an accompaniment to the diving-bell, this telescope would be of great service. Those who descend into the sea might, without quitting the bell, examine the bottom to a considerable distance around them, and thus save a great deal of that trouble and danger which must necessarily attend these submarine operations. In moderate depths, the instrument alone might be used to discover objects at the bottom of the sea; and when the position of these is once ascertained, they might afterwards be brought up either by diving, or by a descent in the diving-bell. In rivers and lakes, it might be employed with the same success in recovering any article that has been lost.

In the interesting pursuits of the geologist and the natural historian, the present instrument will find an important application. It will enable them to examine the nature of rocks and strata in the beds of rivers and lakes; to observe the

phenomena of aquatic plants; to watch the manners and motions of fishes and other subaqueous animals; and thus to combine, in a high degree, the purposes of science and amusement. There is perhaps no recreation more delightful, and certainly none more rational, than to be carried along the surface of a pellucid stream, and to observe all the phenomena below, when the broken surface of the current seems to render it impenetrable to human eyes, and to secure its watery inhabitants from the observation and hostility of man.

In some of the valuable fisheries, this telescope might also be advantageously employed. In fishing for the pinna marina, as it is practised on the coast of Naples; in discovering the rocks which produce the corals, when the depth is moderate; and in finding the banks on which the pearl oysters have taken up their abode—the fishermen might derive essential aid from an instrument of this construction. In the more humble operation of finding the haunts of salmon in rivers, and of enabling the fishermen to direct his spear with a more certain aim, the preceding instrument is not without its use.

In observing the results of experiments that require to be made under water, and in examining the foundations of wooden bridges and of

moles, and the effects produced by the currents on the strata at the piers of stone bridges, the present instrument will be of considerable advantage.

An instrument for viewing objects under water might be applied to a gun, for the purpose of enabling the sportsman to direct it with certainty to any living object at the bottom of a pool or river.

The application of the same principle to the construction of microscopes will be explained in the following Book.

CHAP II.

Description of an Instrument for measuring the Refractive Powers of Fluids, and of a method of determining the Refractive Powers of Solids; with Tables of the Refractive Powers of various Substances.

THERE is perhaps no part of natural philosophy more truly interesting, than that which relates to the determination of the physical properties of bodies. An accurate knowledge of these properties is of extensive use in the arts and sciences, and has conducted the experimental philosopher to some of the finest inventions and discoveries, of which the human mind can boast. In the ardour of research, by which the last century was characterised, investigations of this kind were by no means overlooked, though they were in a great measure confined to the mechanical and chemical properties of opaque bodies. It is only of late years, that philosophers have turned their serious attention to the powers of transparent substances, in refracting and dispersing the rays

of light; and though the improvement of optical instruments is involved in the inquiry, yet this branch of physics must be regarded as still in its infancy. Every attempt, therefore, however humble, of facilitating the determination of refractive and dispersive powers, or of confirming and correcting the results obtained by preceding authors, is entitled to the particular attention, both of chemists and experimental philosophers.

The method of measuring refractive powers, which has been most generally employed, is to form a prism of the transparent substance, and to ascertain the real deviation of a solar ray from its original direction, when transmitted through the two surfaces of the prism. The prism is turned slowly about an axis parallel to the common section of its planes, till the refracted ray is stationary between its two opposite motions; and when this position is obtained, the incident and the emergent rays form equal angles with the surfaces at which they are refracted. The deviation of the solar ray, and the angle contained by the refracting surfaces, being accurately measured, the ratio of the sines of the angles of incidence and refraction may be found by a simple calculation. In order to obtain an accurate measure of this deviation, Sir Isaac Newton applied the prism to a quadrant, and obser-

ved the angle which the mean refrangible rays made with the horizon; and by means of this angle, and the sun's altitude, observed at the same time, he found the angle of refraction, and consequently the ratio of the sines of the angles of incidence and refraction, for the mean refrangible rays.

The celebrated Euler suggested a method of measuring the refractive powers of transparent fluids, by inclosing them between two large meniscuses of glass, and observing the focal length of the compound lens, as altered by the different convex lenses of the fluids which were inclosed. The curvature of the meniscuses, and the refractive power of the glass from which they were made, being accurately determined, the refractive powers of the inclosed fluids were easily ascertained. This method was put in practice by his son Albert Euler; but he applied it only to a very few substances, and obtained no results which were deserving of notice.

Both these methods, however, though sufficiently accurate for transparent fluids, are completely inapplicable to a numerous class of soft and solid substances, which are partly diaphanous; and even to those fluids, such as petroleum, balsam of Peru, balsam of sulphur, &c. which are but imperfectly transparent.

A new and elegant method of examining refractive powers by prismatic reflection, has been recently proposed by Dr Wollaston, who has thus ascertained the index of refraction for more than 50 different substances. " This method," says Dr Wollaston, " was suggested by a consideration of Sir Isaac Newton's prismatic eye-glass, the principle of which depends on the reflection of light at the inner surface of a dense refracting medium. Since the range of inclination within which total reflection takes place, depends not only on the density of the reflecting prism, but also on the rarity of the medium adjacent to it, the extent of that range varies with the difference of the densities of the two media. When, therefore, the refractive power of one medium is known, that of any rarer medium may be learned by examining at what angle a ray of light will be reflected from it.

" For instance, when any object is laid under a prism of flint-glass, with air alone interposed, the internal angle of incidence at which the visual ray begins to be totally reflected, and at which the object ceases to be seen by refraction, is about 39° 10′; but, when the object has been dipped in water, and brought into contact with the glass, it continues visible by means of the higher refrac-

tive power of the water, as far as $57\frac{1}{2}°$ of incidence. When any kind of oil, or any resinous cement, is interposed, this angle is still greater, according to the refractive power of the medium employed; and by cements that refract more strongly than the glass, the object may be seen through the prism, at whatever angle of incidence it is viewed.

"In examining the refractive powers of fluids, or of fusible substances, the requisite contact is easily obtained; but with solids which can in few instances be made to touch to any great extent, this cannot be effected without the interposition of some fluid or cement of higher refractive power than the medium under examination. Since the surfaces of a stratum so interposed are parallel, it will not affect the total deviation of a ray passing through it, and may therefore be employed without risk of any error in consequence."

The instrument which Dr Wollaston has constructed upon this principle, is extremely simple and ingenious, and does great credit to the inventive powers of that distinguished philosopher; but as I have never examined the instrument itself, I am by no means entitled to pronounce upon its accuracy. Dr Thomas Young, however, a very competent judge, has remarked,

that Dr Wollaston's numbers belong correctly to the extreme red rays; and if this observation is well founded, all his measures of refractive powers must be increased by half the angle of dispersion, a quantity which cannot be obtained till the index of refraction has been previously determined. Independent of this objection, I cannot but suspect, that the principle of prismatic reflection is liable, in practice, to some source of error, against which Dr Wollaston has not sufficiently provided. I am induced to hazard this conjecture, by observing the enormous difference between many of his results and mine: a difference which could not possibly have arisen from any inaccuracy of observation, and still less from any variety in the nature of the substances which were examined. Pitch, for example, is ranked by Dr Wollaston in the order of refractive powers as below oil of sassafras, and even below Radcliffe crown-glass, whereas, according to my experiments, it ranks very far above oil of sassafras. This discrepancy appeared to me so great, that I repeated the experiment with different kinds of oil of sassafras and pitch, but I uniformly obtained the same result. The refractive power of phosphorus, which Dr Wollaston makes 1.579, below both horn and flint-glass, affords a still more stri-

king proof of this conjecture. I have examined this substance with particular care, and have found its refractive power to exceed that of any substance which I have tried by this method, and to rank between sulphur and diamond; a result which might have been expected from its high degree of inflammability. In the observations upon the following Table of Refractive Powers, I shall have occasion to resume this subject, and to point out other cases in which the measures seem to be fallacious.

My attention was naturally drawn to the subject of refractive powers, while using the instruments for viewing objects under water, which have been described in the preceding Chapter. It was obvious, that the focal length of the object-glass of a telescope varied with the refractive power of the fluid in which it was plunged; and that, if a compound microscope were employed, where the image is always formed at the same distance behind the object-glass, the distance of the object from the object-glass, or the magnitude of the image measured by a micrometer, would likewise afford a measure of the refractive power of the fluid in which the object and the object-glass were immersed. I accordingly fitted up a compound microscope with a suitable apparatus, and, by pla-

cing an object at the bottom of a glass-vessel containing the fluid to be examined, and immersing the exterior surface of the object-glass in the fluid, the distance between the object and the object-glass furnished a relative, or, by a little calculation, an absolute measure of the refractive power of the fluid. After making a number of experiments in this way, I saw that the principle was not sufficiently general in its application; that a great depth of fluid was necessary when its refractive power was great; and that it could not be employed in the examination of imperfectly transparent fluids, and of the numerous class of soft solid substances, including the various gums and resins. I therefore abandoned it altogether, and adopted the method which I shall now proceed to describe

At the extremity MN (Plate X. Fig. 6.) of a compound microscope, at which the object-glass is placed, a piece of thin parallel glass *a* is fixed in a position perpendicular to the axis of the instrument. A double and equally convex lens *b*, whose axis is also coincident with that of the microscope, is inserted at the end of a small tube of brass ABCD, which screws upon the piece MN, so that the inner surface of the lens can be brought into contact with the piece of glass *a*, or placed at

any small distance from it. By means of two opposite perforations, in the side of the tube ABCD, immediately behind the lens *b*, a small portion of any substance can be introduced between the convex lens and the piece of parallel glass. If the substance is fluid, it will form a plano-concave lens, whose thickness may be diminished to any assignable magnitude, by screwing the lens *b* nearer to the plate of parallel glass; and if the substance is soft, and imperfectly transparent, it may be easily pressed, by the force of the screw, into a plano-concave lens, so extremely thin at the centre as to be completely diaphanous. By this means I have obtained the most perfectly transparent lenses of aloes, pitch, opium, asafœtida, dragons' blood, caoutchouc, and many other substances, through which light had never before been regularly refracted.

The plano-concave lens of the refracting body which is thus formed, obviously tends to increase the focal length of the convex lens *b*, and, consequently, to form the image of any object placed at *m*, at a greater distance than the point P in the anterior focus of the eye-glass QR; but as the lenses QR, LL, and *b*, are all fixed at invariable distances, it becomes necessary to remove

the object to n, in order that a distinct image of it may be formed at P. If a fluid of greater refractive density is interposed between the lenses, the plano-concave lens which it forms will have a greater power in augmenting the focal length of b; and therefore, in order to have a distinct image at P, the object must be removed to some point, as at n, at a greater distance from b. Hence the distances $b\,m$, $b\,n$, &c. when carefully measured, will furnish a relative, or, by a simple calculation, an absolute value of the refractive power of the inclosed fluid. For those who are desirous to have the index of refraction without any trouble, the instrument might be easily fitted up so as to shew this by inspection; the index of refraction, corresponding to several of the distances $b\,m$, $b\,n$, &c. being determined by direct experiment.

In the following experiments, the object viewed by the microscope was a number of minute scratches upon the upper surface of a piece of plane glass; the distances $b\,m$, $b\,n$, &c. were measured by a pair of reverted calipers upon a well divided scale; the lenses were preserved at an invariable distance from each other; the thickness of the plano-concave lens, at the centre, was

kept as uniform as possible; and the greatest care was taken to observe the image which was formed by the mean refrangible rays. In order to prevent any error in judging of the instant of distinct vision, from a variation in the focal length of the eye, a delicate fibre of glass, with a transparent axis, was stretched across the diaphragm, at the anterior focus of the eye-glass. The eye being always accommodated to the central part of this fibre at the moment of observation, it is manifest that all the results were obtained when the eye of the observer had the same focal length.

The numbers in the following Tables represent merely the distances *b m*, *b n*, &c. or the interval between the object and the object-glass of the microscope. I intended, at first, to have computed from these data the index of refraction, or the ratio between the sines of the angles of incidence and refraction for each substance; but, as the object-glass had a very small diameter, and as I did not possess the tools in which it was ground, I could not trust to the value of the radius of its surfaces, as determined merely from the lens itself. To those, however, who may wish to repeat the experiments, the following formulæ will be of considerable service, and will enable them to find

the index of refraction without much trouble. Let

$m=$ index of refraction for the convex lens A, Plate X. Fig. 7.;
$\mu=$ index of refraction for the concave fluid lens B;
$p=\frac{1}{m-1}$, and $\pi=\frac{1}{\mu-1}$;
$d=$ SA, the distance of the object from the object-glass, as given in the following Tables;
$f=$ AF, the focal length of the lens A, for rays diverging from S;
$\varphi=$ Af, the focal length of the compound lens AB;
$r=$ the radius of each surface of the lens A:

Then we shall have,

$$f=\frac{p\,d\,r}{2d-p\,r}$$

$$\varphi=\frac{\pi f r}{f-\pi r}\ *$$

$$\pi=\frac{\varphi f}{\varphi r-f r};\ \text{and,}$$

$$\mu=\frac{\varphi r-f r}{\varphi f}+1.$$

In the object-glass which was employed for the following experiments, r was nearly 1.16; but I was at no trouble in acertaining this number, as a new lens was executing for me, in such a man-

* As φ is a constant quantity, it may be found either by direct observation, or by assuming the index of refraction for water to be 1.336.

ner that the radius of its surfaces might be accurately ascertained.

The determination of the index of refraction is, after all, of very little importance. The numbers in the following Table are sufficient for every purpose of physics and chemistry; and those who wish to employ any of the substances for optical purposes, will be disposed to determine the ratio of the sines for themselves.

TABLE I. *Containing the Refractive Powers of various Solid and Fluid Substances.**

	Diamond, - - -	See page 282.
	Phosphorus, - -	See Table II.
	Sulphur, - - - - -	See Table II.
	Aloes, socotrine, - - - -	5.120
5.	Ditto, Barbadoes, - - -	5.120
	† Oil of cinnamon, No. 12. inspissated by exposure to the air 1 hour, - -	5.087

* It may be proper to mention, that almost all the oils in the following Table were procured from an apothecary's shop, and may not therefore be of the genuine kind.

† It is very singular, that Mr Hauksbee should make the refractive power of oil of cinnamon considerably less than that of oil of sassafras. No other author, so far as I know, has made the experiment; so that, till now, the very unusual refractive and dispersive qualities of this fluid, and of oil of cassia, have been unknown. The oil of cinnamon, No. 6. and 13. were out of the same bottle, and, though sold as oil of cinnamon, it is certainly oil of cassia. It had the appearance of being adulterated with some foreign substance, both from its smell and from its colour, which resembled that of brandy. The oil of cinnamon, No. 9. was from a different shop, and had the same colour with the oil of cassia, No. 7. which was sold under its right name. Perhaps the genuine oil of cinnamon, which I have not been able to obtain, might have been that which Mr Hauksbee employed; though it is highly improbable that its refractive power could be so very low.

	Oil of cassia, - - - -	5.077
	Balsam of Tolu, - - -	4.987
	Oil of cinnamon, (a different kind) -	4.837
10.	* Muriate of antimony after standing 2 days,	4.710
	Resin of jalap, - - -	4.631
	Balsam of Peru, - - -	4.576
	Oil of cinnamon, - - -	4.560
	Guiacum, - - - -	4.498
15.	Muriate of antimony after standing 22 hours,	4.473
	† Pitch, - - - -	4.201
	Ditto, another kind, - - -	4.198
	Ditto, burned a little, - -	4.311
	Gum ammoniac, - - -	4.159
20.	Asafœtida, - - - -	4.106
	Dragon's blood, - - - -	4.009
	Manna burned into a very deep colour,	3.996
	Juice of the *Asarum Europeum*, after standing 18 hours, - - - -	3.949
	Opium, - - - -	3.921

* This muriate of antimony is the same as No. 79. Its refractive power increased very rapidly by exposure to the air; a result which is the more singular, as this substance imbibes water very rapidly from the atmosphere. As the refractive power of water is less than that of any other substance, it uniformly diminishes the refractive power of any body with which it is mixed, and hence we should have expected a result the very reverse of what was obtained. See TABLE V.

† It is very remarkable, that vision was more perfect when a concave lens of pitch was employed, than when it was made of any other substance.

25.	Caoutchouc, - - - -	3.887
	Muriate of antimony after standing 5 hours,	3.847
	Copal, - - - -	3.843
	* Glue, nearly hard, - - -	3.841
	Manna burned into a deeper colour than No. 31. - - - -	3.832
30.	Rosin, - - - - - -	3.831
	Elemi, - - - -	3.811
	Gum galbanum, - - -	3.811
	Manna burned into a yellowish-brown colour,	3.774
	Gum anime - - - -	3.767
35.	Gum Thus, - - - -	3.766
	Burgundy pitch, - - -	3.761
	Turpentine, common, from a wrought plank,	3.757
	White sugar melted by heat, - -	3.753
	Gum sagapenum, - - -	3.751
40.	Chio turpentine, - - -	3.748
	Petroleum, - - - -	3.739
	Benzoin, - - - -	3.722
	Gum juniper, - - -	3.711
	Oil of cinnamon and oil of olives, 1 part each,	3.692
45.	Oil of cloves, - - - - -	3.688
	Mastic, - - - - - -	3.678
	Oil of anise seeds, - - -	3.657

* This glue was placed between the glasses when it was in such a state of induration that it could scarcely be cut with a knife. It hardened by exposure to the air, but was not altogether free from water when the experiment was made.

	Oil of sassafras, - - - -	3.651
	Manna slightly heated between the lenses, and then cooled, - - - - -	3.623
50.	Canada balsam, - - -	3.617
	Gum olibanum, - - - -	3.610
	Juice of the *Urtica dioica*, after standing some time, (not a good observation,) -	3.592
	* Genuine balsam of Gilead, - -	3.580
	Shell lac, - - - -	3.573
55.	Resin fresh from the larch, - -	3.567
	Resin of the bile of an ox, (from John Davy, Esq.) - - - -	3.567
	Chio turpentine melted, - -	3.560
	Oil of mace, No. 123. cooled, - -	3.547
	Oil of Barbadoes tar, - - -	3.526
60.	Skimmed milk mixed with water, and inspissated by evaporation, - -	3.520
	Gum myrrh, - - - -	3.465
	Glue as soft as caoutchouc, - -	3.458
	Balsam of Capivi, - - - -	3.457
	Oil of cinnamon 1 part, oil of olives 2 parts,	3.443
65.	Juice of a ripe orange inspissated, -	3.433
	Gum Arabic, not altogether free from water,	3.423
	Oil of mace, - - - - -	3.413
	Weak infusion of senna, after standing 9 hours,	3.412
	Juice of the *Sedum Telephium*, after standing 14 hours, - - - - -	3.412

* In the possession of John Murray, Esq. and brought from Mecca by Captain Vashon.

70. Juice of the *Angelica Archangelica*, after standing several hours, - - -	3.402
Juice of the *Leontodon taraxacum*, after standing 14 hours, - - - -	3.400
Juice of the *Lactuca virosa*, after standing 10 hours, - - - -	3.400
Scammony, - - - - - -	3.400
Juice of the *Sanguinaria Canadensis*, after standing 12 hours, - - - -	3.387
75. Oil of sweet fennel seeds, -	3.376
White wax melted, and then cooled, -	3.375
Oil of amber, - - - -	3.373
Starch dried, - - - -	3.347
Muriate of antimony before exposure to the air,	3.347
80. Orange juice, after standing 18 hours, -	3.347
Juice of the *Ranunculus Flammula*, after standing 7 hours, - - - - -	3.337
Juice of the *Angelica sylvestris*, after standing 4½ hours, - - - - -	3.334
Oil of pimento, or Jamaica pepper, -	3.334
Oil of Rhodium, - - - -	3.333
85. Spermaceti, cold, - - - -	3.318
Hemlock juice (*Conium maculatum*), after standing 6^h 50′, - - -	3.317
Yolk of an egg nearly dried, - -	3.310
Treacle, - - - -	3.307
Oil of cinnamon 1 part, oil of olives 4 parts,	3.283
90. Camphor, - - - -	3.280
Oil of spearmint, - - -	3.277

	Oil of hyssop, - - - - -	3.271
	Honey, - - - -	3.267
	Balsam of sulphur, - - -	3.259
95.	Bees wax, cold, - - -	3.243
	Tallow, cold, - - - - -	3.243
	Yolk of an egg, after 15 hours exposure,	3.234
	Genuine oil of juniper, (from John Murray, Esq.) - - - - - - -	3.231
	Oil of nutmeg, - - -	3.227
100.	Oil of caraway seeds, - - -	3.223
	Oil of penny royal, - - -	3.220
	Oil of lemon, - - -	3.216
	Hemlock juice, after standing 4h 25′,	3.210
	Oil of wormwood, yellow coloured, after standing 6 hours, - -	3.210
105.	Alum, - - - - - -	3.209
	Raspberry jam, - - -	3.207
	Oil of dill seed, - - - -	3.201
	Windsor soap, - - - -	3.200
	Lintseed oil, - - - -	3.196
110.	Orange juice, after standing 8 hours, -	3.196
	Oil of thyme, - - -	3.190
	Oil of cinnamon 1 part, oil of olives 8 parts,	3.187
	Oil of savine, - - -	3.184
	Florence oil, - - - -	3.183
115.	Castor oil, - - - -	3.183
	Oil of wormwood, yellow coloured, -	3.181
	Oil of laurel, (a plaster for horses feet,)	3.170
	Tallow, melted, - - -	3.167
	Train oil, - - - - - -	3.167

120.	Oil of juniper, - - - -	3.157
	Milk of the cocoa nut, after standing 8 hours,	3.156
	Oil of almonds, - - -	3.153
	Oil of mace, No. 67. melted, - -	3.147
	Naples soap, - - - - -	3.137
125.	Cajeput oil, - - - - -	3.126
	Oil of cinnamon 1 part, oil of olives 12 parts,	3.120
	Huile antique de la rose, - - -	3.116
	Oil of turpentine, - - - -	3.115
	Oil of camomile, - - -	3.114
130.	Oil of olives, - - - - -	3.113
	Juice of a ripe orange, after standing 4 hours,	3.107
	Oil of lavender, - - -	3.105
	Naphtha, - - -	3.105
	Rapeseed oil, or green oil, - -	3.105
135.	Oil of palm, - - - -	3.103
	Fresh butter, - - - - -	3.098
	Spermaceti oil, - - - -	3.090
	Oil of peppermint, - - -	3.089
	Oil of bergamot, - - -	3.088
140.	Oil of rosemary, - - -	3.077
	Interior of the crystalline of a haddock, (not the nucleus,) - - -	3.067
	Oil of brick, distilled from spermaceti oil,	3.066
	Salt butter, - - - -	3.053
	Marmalade, - - -	3.047
145.	Jelly from lamb, after standing 15 hours,	3.047
	Yolk of an egg, after a minute's exposure to the air, - - - - -	3.041

	Tallow, melted, - - - -	3.038
	Juice of the *Rumex sanguineus*, after standing several hours, - - - - -	3.037
	White wax, melted, - - -	3.003
150.	Spermaceti, melted, - - -	2.946
	Bees' wax, melted, - - -	2.94
	Oil of rhue, - - - -	2.909
	Sulphuric acid of the shops, - -	2.867
	* Muriate of antimony, - - -	2.853
155.	External part of the crystalline of a young haddock, - - -	2.843
	Phosphorous acid, - - -	2.833
	Central portion of the crystalline lens of a lamb, - - - -	2.829
	Middle coat of do. do. - -	2.780
	Yolk of a hen's egg, (newly taken out,)	2.778
160.	Gluten of wheat, dried between the lenses,	2.767
	† Gum dragon, (nearly dry,) - -	2.725
	Sulphuric acid, No. 153. after standing half an hour in damp air, - -	2.687

* This is the muriate of antimony of the shops, and is much inferior in refractive power to No. 79.

† Dr Wollaston makes the refractive power of gum-dragon between 1.657 and 1.768, which is very much greater than balsam of Tolu, whereas, in the preceding Table, it is lower than all the balsams, gums, resins, and oils. The portion of this substance, however, which I used, was not altogether dry, so that its refractive power in the preceding Table is lower than it would have been, had the water which it contained been completely evaporated. See the TABLE, p. 286, No. 95.

	External part of the crystalline of a haddock, (older than No. 155.) - -	2.670
	Crystalline of a pigeon's eye, - -	2.650
165.	Juice of the rhind of a ripe orange, -	2.633
	* Pus, - - - - - -	2.587
	Essence of peppermint, - - -	2.577
	Aromatic spirit of vinegar, - -	2.555
	Milk of the cocoa nut, - -	2.547
170.	Outer coat of the crystalline of a lamb, (see No. 157, 158.) - - -	2.541
	Cornea of ditto, - - -	2.541
	Juice of the fruit of a ripe orange, -	2.517
	Oil of wine, - - -	2.504
	Oil of ambergrease, - - -	2.504
175.	Alcohol, - - - -	2.497
	Whitish fluid between the crystalline and its capsule, in the haddock No. 155. -	2.491
	Fluid from the crystalline of a lamb, after puncturing the capsule, (see No. 170.)	2.473
	Bile of a fowl, - - - -	2.473
	Juice of the *Sonchus oleraceus*, - -	2.473
180.	Ink, - - - -	2.467
	Ketchup, - - - - -	2.460
	Juice of the *Chelidonum majus*, -	2.448
	Juice of the *Angelica Archangelica*, -	2.447
	Strong Highland whisky, -	2.446

* The difference of the refractive power of pus and mucus is so very considerable, that the one could scarcely be mistaken for the other when this optical test is employed.

185.	Laudanum,	2.446
	Essence of peppermint,	2.436
	Juice of the *Asarum Europeum*,	2.433
	Arquebuzade,	2.422
	Brandy,	2.413
190.	Rum,	2.413
	White of a hen's egg,	2.409
	Juice of the *Leontodon taraxacum*, or dandelion,	2.403
	Juice of the *Ranunculus Flammula*,	2.399
	Juice of the *Sanguinaria Canadensis*,	2.398
195.	Juice of the *Urtica dioica*,	2.397
	Oil of boxwood,	2.396
	Juice of the *Angelica sylvestris*,	2.393
	Jelly from cold lamb,	2.393
	Juice of the fruit of a ripe orange newly taken out,	2.392
200.	Juice of the *Conium maculatum*, or hemlock, newly taken out,	2.390
	Human blood,	2.387
	Juice of the *Sedum Telephium*,	2.387
	Vitreous humour of a pigeon's eye,	2.380
	Port wine,	2.373
205.	Strong infusion of tea,	2.357
	Juice of the *Lactucà virosa*,	2.354
	Weak infusion of senna,	2.353
	Vinegar,	2.347
	Vitreous humour of a lamb's eye,	2.346
210.	Juice of the *Rumex sanguineus*,	2.343
	Aqueous humour of a haddock's eye,	2.326
	Vitreous humour of do.	2.326

Expectorated mucus, - - -	2.321
Saliva, - - - - - - -	2.321
215. Water, - - - -	2.309
Air, - - - - -	1.425

When sulphur was placed between the lenses, its refractive power was so great, that the refraction at one concave surface of this substance was nearly equal to the refraction at two surfaces of glass of the same convexity. Hence the compound lens, instead of being convex, almost resembled a piece of parallel glass in which the refractions are equal and opposite.

In subjecting phosphorus to a similar trial, I was led to expect, from Dr Wollaston's measure of its refractive power, a very different result; but I was surprised to find its refractive density so enormous, that the deviation produced at one concave surface of this substance, was greater than the two refractions at both the convex surfaces of the lens, and that the compound lens had actually become concave. The refractive power, therefore, of these two inflammable substances, extended far beyond the scale of Table I. so that I was under the necessity of employing a new object-glass for the microscope, having its sides unequally convex, and its flat surface turned

to the piece of plane glass. By this means, the concave surface of the sulphur or phosphorus had such a small degree of curvature, that it was incapable of counteracting the deep convexity of the exterior surface of the lens of glass; and a compound lens was thus obtained which had always a positive focus, and which required the object to be placed at a distance sufficiently commodious for the purposes of observation.

With this new object-glass I obtained the following results for sulphur and phosphorus, and several other bodies; and, in order to shew the results nature of the new scale, I have added the results for a few other substances which are already included, upon a different scale, in Table I.

TABLE II. *Containing the Refractive Powers of Phosphorus, Sulphur, &c.*

	Refraction from air, - - -	1.000
	Water, - - - - -	1.345
	Ether, - - - -	1.400
	Alcohol, - - - -	1.404
5.	Tincture of cantharides, - -	1.413
	Muriatic acid, - - - -	1.431
	Nitrous acid, - - - -	1.446
	Nitric acid, - - - -	1.456

	Hydrate of soda melted by heat,	1.458
10.	* Hydrophosphoric acid melted and hot,	1.476
	Do. when cold,	1.507
	† Compound of chlorine and manganese after deliquescence,	1.500
	Do. after standing all night,	1.516
	Sulphuric acid,	1.517
15.	Oil of poppy,	1.584
	Oil of turpentine,	1.588
	Oil of feugreck,	1.593
	Oil of marjoram,	1.596
	Nut oil,	1.600
20.	Oil of angelica,	1.600
	Solution of gum-kino in alcohol, just transparent,	1.600
	Bird-lime,	1.630
	Oil of pimento,	1.637
	Balsam of capivi,	1.646
25.	Oil of Cumin,	1.650
	Oil of sassafras,	1.663
	Oil of the cashew nut,	1.692
	Sugar after fusion,	1.704
	Rosin,	1.720
30.	Pitch,	1.806
	Oil of cinnamon,	1.817

* Made by Sir Humphry Davy, and given to me by John Davy, Esq.

† From John Davy, Esq. See *Phil. Trans.* 1812.

	Balsam of Peru, - - - - -	1.826
	Balsam of Tolu, - - - -	1.871
	Castor from the beaver, - -	1.900
35.	Oil of cassia, - - - -	1.911
	Sulphur, - - - - -	4.337
	Phosphorus, - - - -	7.094

This measure of the refractive power of phosphorus, so extremely different from the result obtained by Dr Wollaston,* confirms the beautiful and sagacious conjecture of Sir Isaac Newton, that all inflammable substances have high refractive powers. This conjecture, founded on a very limited number of experiments, was completely overturned by the results of Dr Wollaston's observations; and I therefore feel peculiar satisfaction in restoring to credit the opinion of our immortal countryman, and in establishing, for the first time, that the *refractive powers of the three simple inflammable substances are in the very order of their inflammability.*

In making the experiment with phosphorus, I was particularly solicitous to guard against every source of error, and I have now repeated it more

* The result obtained by this ingenious philosopher, when reduced to the scale in the preceding Table, is about 1.8 instead of 7.094.

than six times with the same result.* Some difficulty will be experienced, by those who choose to repeat the experiment, in moulding a thin plate of phosphorus into a plano-concave lens. The phosphorous acid, which is instantly formed upon its surface by exposure to the air, must be carefully taken off by a piece of bibulous paper before the substance is placed between the lenses.

In order to find the index of refraction when the lens of glass is unequally convex, let us suppose r to be radius of the surface next the object, and R the radius of the other surface, and, consequently, the radius of the plano-concave lens of fluid; then, the other quantities being indicated by the same letters, as in page 251. we shall have

$$f=\frac{p\,d\,r\,R}{d\,r+d\,R-p\,r\,R}$$

$$\varphi=\frac{\pi f R}{f-\pi R}$$

$$\pi=\frac{\varphi f}{\varphi R-f R}$$

$$\mu=\frac{\varphi R-f R}{\varphi f}+1.$$

* It will be seen from the Table in p. 283. that I have obtained a similar result by forming the phosphorus into prisms between plates of glass.

Though all the following Tables, except Table V. have already been given in the General Table, yet it is of importance to have them also in a separate form. The scale is precisely the same as in Table I.

TABLE III. *Containing the Refractive Powers of the Fluids of a* YOUNG HADDOCK'S *Eye.*

Aqueous humour, - - - - -	2.326
Vitreous humour, - - -	2.326
Whitish fluid between the crystalline and its capsule, - - - - -	2.491
External part of the crystalline, - -	2.845
The central part of the crystalline, 0.133; of an inch thick, placed between the lenses, -	0.377

The central part of the crystalline, in the last of these experiments, was rolled between the finger and the thumb till it was deprived of all the softer parts, and till only a small hard nucleus remained, having a diameter of 13 hundredths of an inch. This nucleus was placed between the lenses, but the distance of the object, instead of being about 3.00 inches, as one would, without much thought, have expected, was only 37 hun-

dredths of an inch. This curious result is a complete proof of the great rapidity with which the refractive density of the lens increases towards the centre, and will be understood from Plate X. Fig. 8. where CD is the piece of parallel glass, AB the convex object-glass, and EF the nucleus of the crystalline. Now, since the concave surface, which is in contact with the posterior surface of the lens AB, is obviously formed by the exterior coat *m m* of the crystalline; and since this coat has a less refractive power than the interior nucleus *n*, this nucleus must act as a concave lens; and its action is so powerful, that it far more than counteracts the refraction produced at the concave surface of *m m*. Had the refraction of the external nucleus been precisely equal to the refraction of the concave surface of the crystalline, the distance of the object would have been 2.843 inches instead of 0.377. It must be recollected, however, that the nucleus *n* refracts like a double convex lens, while the coat *m m* refracts only at its anterior surface.

TABLE IV. *Refractive Powers of the Fluids of a Lamb's Eye.*

Vitreous humour, - - - -	2.346
Fluid in the crystalline after puncturing the capsule, - - - - -	2.475
Outer coat of the crystalline lens, - -	2.541
Middle coat of do. - - -	2.780
Central portion of do. - - -	2.829

The following experiments were made upon muriate of antimony, in order to ascertain the cause of its refractive density being increased by exposure to the air. The scale is the same as in Table II.

TABLE V. *Refractive Power of Muriate of Antimony.*

Muriate of antimony, - - -	1.600
Do. exposed to the air, - - - -	1.642
Do. exposed to the air longer, - -	1.700
Do. exposed to the damp open air, -	1.578
Do. taken into the room, - - -	1.643
Do. after being placed in dry air, - -	1.687
Do. being exposed to the light of the sun, -	1.750
Do. do. - - -	1.800

Do. being exposed to the light of the sun,	-	1.927
Do.	do. the sun's light being very weak,	1.850
Do.	do. the sun's light being weaker,	1.827
Do.	do. placed in damp air, -	1.667

Table VI. *Refractive Powers of Vegetable Juices.*

1. Juice of the fruit of a ripe orange newly taken out, - - -	2.392
Do. after standing several days, - -	3.433
2. Juice of the *Conium maculatum*, or common hemlock, - - - - -	2.390
Do. after standing $6^h\ 50'$, - -	3.317
3. Juice of the *Angelica sylvestris*, - -	2.393
Do. after standing 2 hours, - -	2.833
Do. after standing 4½ hours, - -	3.334
4. Juice of the *Angelica Archangelica*, -	2.447
Do. after standing several hours, -	3.402
5. Juice of the *Sanguinaria Canadensis*, -	2.398
Do. after standing 12 hours, - -	3.387
6. Juice of the *Leontodon taraxacum*, -	2.403
Do. after standing 14 hours, - -	3.400
7. Juice of the *Lactuca virosa*, - -	2.354
Do. after standing 10 hours, - -	3.400
8. Juice of the *Rumex sanguineus*, - -	2.343
Do. after standing some hours, -	2.833
Do. after standing longer, - - -	3.037
9. Juice of the *Chelidonium majus*, - -	2.448

10.	Weak infusion of senna, - -	2.353
	Do. after being exposed to the air 9 hours,	3.412
11.	Juice of the *Asarum Europeum* - -	2.433
	Do. after standing several hours, -	3.648
	Do. after standing longer, - -	3.815
	Do. after standing 18 hours, - -	3.949
12.	Juice of the *Ranunculus Flammula*, -	2.399
	Do. after standing 7 hours, - -	3.357
13.	Juice of the *Sedum Telephium*, - -	2.387
	Do. after standing 14 hours, - -	3.412
14.	Juice of the *Urtica dioica*, - -	2.397
	Do. after standing,—(not a good observation)	3.592
15.	Juice of the *Sonchus oleraceus*, - -	2.475
	Do. after standing 7 hours, - -	3.400
16.	Juice of the *Fragaria Vesca*, -	2.390

The preceding experiments on the refractive powers of vegetable juices, exhibit a singular coincidence of results. The refractive power of them all exceeds a little that of water; and when the aqueous parts are evaporated, the residuum, with a few exceptions, has nearly the same refractive density.

Method of Measuring the Refractive Powers of Solid Fragments.

The preceding method of measuring refractive powers is applicable only to fluid bodies, or to those substances which may be formed into a concave lens by heat, pressure, or evaporation. The various kinds of glass, however, and the numerous class of transparent minerals, are obviously excluded from this method of measurement. In order to determine the refractive power of these hard solids, it is necessary to form them into a prism, having at least two surfaces accurately plane and well polished; to measure the angle of these surfaces, and to calculate the refractive power of the prism from the observed deviation of a transmitted ray of light. The trouble and expence of such a process have prevented it from being frequently employed; and the small number of solids whose refractive powers have been ascertained, is too satisfactory a proof of the difficulties which attend this mode of observation. Even in determining the refractive power of different kinds of flint-glass for achromatic telescopes, the practical optician does not encounter the labour of forming them into prisms, but resorts to the

easy, though inaccurate, method of estimating the index of refraction from the specific gravity of the glass.

A simple and accurate method, therefore, was still wanting for determining the refractive power of solid substances; and I was peculiarly anxious to discover some way in which this object could be attained, without grinding or polishing any part of the solid, and even when its surface was so irregular, either from fracture or from any other cause, that no object whatever could be perceived through the specimen.

It occurred to me, that if a broken chip of any transparent solid were immersed in a fluid of the same refractive power, the incident rays would suffer no refraction in passing from the fluid into the solid, or from the solid into the fluid; and, consequently, that objects could be seen distinctly through the broken chip, whatever was the irregularity either of its form or of its surface. Though this reasoning appeared correct in theory, I scarcely expected in practice so singular a result. In order to make the experiment under the most unfavourable circumstances, I took a piece of crown glass, of a very irregular shape, and so broken in its surface as to appear almost opaque, and, upon plunging it in Canada balsam, I was

surprised to find that it became nearly invisible in the fluid, and that there was so small a refraction of the rays at the confines of the solid and the fluid, that I could even read with facility through all its superficial irregularities.

In this case, the crown glass and the Canada balsam had nearly the same refractive power; and, by increasing the distance of the object, any remaining refraction at the confines of the solid and the fluid was easily detected. By mixing, therefore, fluids of different refractive powers, it was easy to obtain a compound which had the same refractive density with the solid. There was still wanting, however, some exact indication of the precise instant when all refraction was extinguished at the confines of the two media, as it is only in this particular state of things that the refractive power of the fluid could be regarded as a measure of the refractive power of the solid. In order to obtain this indication, I placed between the glasses *a*, *b*, Plate X. Fig. 6. a portion of the fluid which came nearest to the solid in refractive density, and I measured the distance *b n*, at which an object at *n* was seen distinctly. A small chip of the solid was then interposed between the lenses along with the fluid, so that the rays diverging from *n* were transmitted through

the solid. If the distance *b n*, at which the object was seen with perfect distinctness, was now the same as before, it is obvious, that the solid and the fluid had exactly the same refractive power; but, if there was any difference between these distances, the refractive power of the fluid was altered, till the object was seen distinctly at the distance *b n*, both when the refraction was made through the solid alone, and when it was made through the solid and the fluid combined. When this happens, the distance *b n* is a measure of the refractive power of both.

The fluid which I found most convenient for this purpose, was a mixture of oil of cassia and oil of olives, by means of which I could determine the refractive powers of all solids from 5.077, the refractive power of oil of cassia, to 3.113, the refractive power of oil of olives. The following Table will shew pretty nearly the variation of refraction arising from the mixture of these two oils.

Oil of cassia, the same as No. 7. See p. 253.				5.077
Oil of cassia, the same as No. 13. - -				4.560
Oil of cassia 1 part, oil of olives 1 part, -				3.692
Ditto	1 part,	ditto	2 parts,	3.443
Ditto	1 part,	ditto	4 parts,	3.283
Ditto	1 part,	ditto	8 parts,	3.187

Oil of cassia 1 part, oil of olives 12 parts, -	3.120
Oil of olives alone, - - - - - -	3.113

As a specimen of this method of measuring refractive powers, I wished to have given, in the present Chapter, a series of results for the solid substances which I have submitted to examination, but as my experiments are not yet completed, I must reserve this Table for some future occasion.

Before we leave this part of our subject, it may be proper to notice one application of the preceding principle, which promises to be of some practical importance. In ascertaining the soundness of valuable minerals, that have a rough and unpolished surface, the artist is guided by no rules upon which he can rely; and the less experienced purchasers are still more unfit for such a determination. I have often seen specimens of topaz, sold as sound, where the flaws and imperfections were concealed by the ruggedness of their surface, and were not detected till they were wrought by the lapidary. In such cases, we have only to immerse the stone in Canada balsam, oil of sassafras, or any other fluid of nearly the same refractive density, and turn it round with the hand, so that the rays of light may pass through it in every direction. By this means, the slightest flaws,

or cracks, will be instantly perceived, in consequence of the changes which they produce upon the transmitted light. If the stone had been examined in water, the flaws would have been more perceptible than when it was viewed in air; and the distinctness with which they are seen will increase, as the refractive power of the fluid approaches to that of the solid. Hence, in the case of diamond, jargon, spinelle ruby, and other precious stones, whose refractive density exceeds that of any fluid, their imperfections may be detected by immersing them in oil of cassia, or muriate of antimony, though a considerable degree of refraction will still remain at the confines of the solid and the fluid.

The same principle furnishes us with a very simple method of distinguishing many of the precious stones from artificial pastes, which have sometimes been imposed upon the most skilful mineralogists. As the refractive powers of diamond, jargon, ruby, garnet, pyrope, sapphire, tourmaline, rubellite, pistazite, axinite, cinnamon stone, chrysoberyl, and chrysolite, exceed that of oil of cassia, this fluid is particularly applicable to this kind of observation. If any object is viewed through two polished and inclined surfaces, of any substance supposed to be one of

these minerals, when plunged in oil of cassia, the substance will be merely a paste if the refraction is from the point to which the surfaces are inclined, and will be a real mineral if the refraction is made towards that point.

The same method may be advantageously employed by the practical optician, for ascertaining the soundness and purity of the glass which he manufactures into lenses and prisms. There is, perhaps, no kind of labour more frequently wasted, than that which is employed in the formation of lenses and prisms of flint-glass. No sooner are the surfaces polished, than innumerable flaws and veins make their appearance, which the artist was before unable to discover, and which completely distort the image that is formed. A flint-glass prism, indeed, without veins and imperfections, is scarcely to be met with; and the amateur, who has tried to amuse himself in grinding the lenses for achromatic telescopes, must, for the same reason, have found it an impracticable attempt.

In the formation of prisms for measuring the refractive and dispersive powers of bodies that are incapable of receiving a good polish, I have found the same principle of very essential advantage. By cementing upon the two refracting

planes pieces of parallel glass, with a fluid of nearly the same refractive density, substances like horn, tortoise shell, alum, rock salt, and several of the gums, may be rendered perfectly transparent.

In obtaining the measures of dispersive power which are given in the following Chapter, it became necessary to have the index of refraction for every substance which was submitted to examination. I was therefore obliged to institute a new set of experiments for this purpose; and in order to avoid, as much as possible, every source of error that might affect the dispersive powers, I measured the index of refraction by means of the very same prisms in which the dispersion was corrected. In the course of these investigations, I have been led to several results of a very unexpected kind, and, in particular, to the discovery of substances which have a higher refractive power than the diamond. This gem, equally remarkable by its chemical composition and its physical properties, has, since the time of Sir Isaac Newton, who first measured its action upon light, been placed at the head of every table of refractive powers; and it has never even been conjectured, that there existed any other

substance which possessed this optical property in such a high degree. It will appear, however, from the following Table, that Realgar, which is a compound of arsenic and sulphur, and Chromate of lead, or the red lead ore of Siberia, exceed the diamond in their action upon light; the index of refraction for the latter being 2.44, while that of chromate of lead is 2.50, and that of realgar 2.55.

Although the double refraction of chromate of lead is not taken notice of by Hauy, or any other mineralogist, yet I have found it to possess this property in such a remarkable degree, as to be more than triple that of Iceland crystal. While the refractive power of the least refraction is 2.50, that of the greatest refraction amounts to 2.97; and if we compute the dispersive power of the latter, it will be found, that the refractive power of the blue rays is nearly equal to 3.5;—a result so extraordinary, that I felt it necessary to confirm it by various observations made with different crystals of this mineral.

The three substances, therefore, of Chromate of lead, Realgar, and Diamond, may be ranked at the head of those bodies which exercise a particular action upon light. The diamond is distinguished by its extreme brilliancy, by its

property of imbibing light, and by its high refractive power. Realgar is remarkable for a still greater refractive power, and for a power of dispersion above all other bodies but chromate of lead; while chromate of lead possesses the greatest refractive power, the greatest double refraction, and the highest dispersive power, of any substance that has hitherto been tried.

Though many of the other substances contained in the following Table have never before been examined, yet there is nothing singular or unexpected in the results. The precious stones have, in general, a very high refractive power; and the effects of the different metals in altering the refractive power of glass, may be obtained from the results for the various kinds of artificial pastes. The influence of the fluoric acid in diminishing the action of bodies upon light, may be deduced from the refractive powers of fluor spar and cryolite, which are lower than those of any other mineral, or solid body. The last of these substances, which contains more of the fluoric acid than the former, has its refractive density as low as that of salt water; and both these minerals stand at the bottom of the Table of Dispersive Powers.

Table of Refractive Powers.

		Index of Refraction.
	Chromate of lead, greatest refraction, -	2.974
	Do. do. do. do. another kind,	2.926
	Realgar, - - - - - -	2.549
	Chromate of lead, least refraction, -	2.503
5.	Do. do. do. do. another kind,	2.479
	Diamond, brown coloured, - - -	2.487
	Diamond, a different one, - - -	2.470
	Diamond, according to Sir Isaac Newton,	2.439
	Phosphorus, - - - - -	2.224
10.	Glass of antimony, - - - -	2.216
	Sulphur, melted, - - - - - -	2.148
	Sulphur, native, (double refraction,) -	2.115
	Carbonate of lead, double, greatest refract.	2.084
	Carbonate of lead, double, least refraction,	1.813
	Jargon, double, greatest refraction,	2.015
	Jargon, double, least refraction, -	1.961
15.	Sulphate of lead, - - - -	1.925
	Garnet, - - - - - -	1.815
	Blue sapphire, - - - -	1.794
	Pyrope, - - - - - -	1.792
	Jargon (orange-coloured) - - -	1.782
20.	Rubellite, - - - - -	1.779
	Spinelle ruby, - - - - -	1.761
	Chrysoberyl, - - - - -	1.760
	Cinnamon stone, - - - -	1.759

		Index of Refraction.
	Axinite, - - - - - - -	1.735
25.	Deep red-coloured glass, - - -	1.729
	Epidote, double, greatest refraction,	1.703
	Epidote, double, least refraction, -	1.661
	Boracite, - - - - - - -	1.701
	Carbonate of Strontites, double, greatest refraction,	1.700
	Carbonate of Strontites, double, least refraction,	1.543
	Orange-coloured glass, - - - -	1.695
30.	Chrysolite, double, greatest refraction,	1.685
	Chrysolite, double, least refraction,	1.668
	Tourmaline, - - - - - - -	1.668
	Calcareous spar, double, greatest refraction,	1.665
	Calcareous spar, double, least refraction, -	1.519
	Sulphate of barytes, double, greatest refraction,	1.664
	Spargel stone, - - - - - -	1.657
35.	Red topaz, - - -	1.652
	Glass, hyacinth red, - - - -	1.647
	Sulphate of strontites, - - -	1.644
	Oil of cassia, - - - - -	1.641
	Yellow topaz, - - - - -	1.638
40.	Blue topaz from Aberdeenshire, (double refraction) - - - - -	1.636
	Opal-coloured glass, - - - -	1.635
	Balsam of Tolu, - - - -	1.628
	Castor, - - - - - -	1 626
	Muriate of ammonia, - - - -	1.625
45.	Bluish topaz from Cairngorm, - -	1.624
	Guiacum, - - - - - -	1.619

		Index of Refraction.
	* Flint glass, - - - - -	1.616
	Green-coloured glass, - - - -	1.615
	Purple-coloured glass, - - -	1.608
50.	Flint glass, another kind, - - -	1.604
	Red glass, supposed to have been an oriental ruby, - - - - - -	1.601
	Oil of anise seeds, - - - -	1.601
	Beryl, - - - - - -	1.598
	Balsam of Peru, - - - -	1.597
55.	Flint glass, a third kind, - - -	1.596
	Gum ammoniac, - - - -	1.592
	Tortoise shell, - - - - -	1.591
	Emerald, - - - - -	1.585
	Balsam of styrax, - - - -	1.584
60.	Bottle-glass, - - - - -	1.582
	Tartaric acid, double, greatest refraction,	1.575
	Tartaric acid, double, least refraction,	1.518
	Glass, pink-coloured, - - -	1.570
	Horn, - - - - - -	1.565
	Rock crystal, (double) - - -	1.562
65.	Amethyst, - - - - -	1.562
	Gum mastic, - - - - -	1.560
	Burgundy pitch, - - - -	1.560
	Rosin, - - - - - -	1.559

* The refractive powers of the different kinds of flint-glass tried by Boscovich, were 1.590, 1.593, 1.594, 1.604, and 1.625.

		Index of Refraction.
	Chio turpentine, - - - - -	1.557
70.	Rock salt, - - - - - -	1.557
	Sugar, after being melted, - -	1.555
	Gum Thus, - - - - -	1.554
	Chalcedony, - - - - - -	1.553
75.	Sulphate of copper, double, greatest refraction,	1.552
	Sulphate of copper, double, least refraction,	1.531
	Copal, - - - - - -	1.549
	Canada balsam, - - - -	1.549
	Elemi, - - - - - -	1.547
	Olibanum, - - - - -	1.544
80.	Phosphoric acid, solid, - - -	1.544
	Crown glass, - - - - -	1.544
	Gum juniper, - - - - -	1.538
	Selenite, double, greatest refraction, -	1.536
	Feldspar, - - - - - -	1.536
85.	Crown glass, a different kind, - -	1.534
	Caoutchouc, - - - - -	1.534
	Oil of sassafras, - - - - -	1.532
	Glass, coloured, supposed to have been cinnamon stone, - - - - -	1.530
	Balsam of capivi, - - - -	1.528
90.	Leucite, - - - - - -	1.527
	Plate glass, - - - - -	1.527
	Citric acid, - - - - -	1.527
	Shell lac, - - - - -	1.525
	Gum myrrh, - - - - -	1.524
95.	Gum Dragon, - - - - -	1.520

		Index of Refraction.
	Gum Arabic,	1.512
	Sulphate of potash,	1.509
	Oil of cummin,	1.508
	Stilbite,	1.508
100.	Nut oil,	1.507
	Oil of Pimento,	1.507
	Oil of sweet fennel seeds,	1.506
	Oil of Rhodium,	1.505
	Balsam of sulphur,	1.497
105.	Sulphate of iron, double, greatest refraction,	1.494
	Oil of angelica,	1.493
	Oil of marjoram,	1.491
	Oil of caraway seeds,	1.491
	Castor oil,	1.490
110.	Obsidian,	1.488
	Oil of hyssop,	1.487
	Oil of feugreck,	1.487
	Cajeput oil,	1.483
	Oil of almonds,	1.483
115.	Oil of savine,	1.482
	Oil of penny royal,	1.482
	Oil of lemon,	1.481
	Oil of spearmint,	1.481
	Oil of thyme,	1.477
120.	Oil of dill seed,	1.477
	Oil of turpentine,	1.475
	Rapeseed oil,	1.475
	Borax,	1.475

		Index of Refraction.
	Oil of juniper,	1.473
125.	Oil of brick,	1.471
	Oil of Bergamot,	1.471
	Oil of olives,	1.470
	Spermaceti oil,	1.470
	Oil of rosemary,	1.469
130.	Oil of poppy,	1.463
	Oil of lavender,	1.457
	Oil of chamomyle,	1.457
	Oil of wormwood,	1.453
	Hydrophosphoric acid,	1.442
135.	Sulphuric acid,	1.440
	Fluor spar,	1.436
	Oil of rhue,	1.433
	Nitric acid,	1.406
	Nitrous acid,	1.396
140.	Muriatic acid,	1.376
	Alcohol,	1.374
	Oil of ambergrease,	1.368
	White of an egg,	1.361
	Jelly fish, (*Medusa æquoria*)	1.345
145.	Cryolite,	1.344
	Salt water,	1.343
	Water,	1.336
	*Ice,	1.307

* During the melting of the ice, the image which was formerly seen through it with great distinctness, became quite invisible till the ice was completely converted into water.

CHAP. III.

Description of an Instrument for Measuring the Dispersive and Refractive Powers of Solid and Fluid Substances; with Remarks on the irrationality of the coloured Spaces in different media; and a Table of the Dispersive Powers of various Bodies.

A VARIABLE prism, or a prism in which the refracting angle can be varied at the pleasure of the observer, has been long regarded as one of the greatest desiderata in the science of optics.*

Both Clairaut and Boscovich, two of the most distinguished writers on the subject of achromatic telescopes, have constructed and employed a prismatic instrument of this kind, for measuring refractive and dispersive powers; but none of these contrivances possessed those requisites of simplicity and accuracy, which could alone recom-

* *Adhuc tamen* (says Boscovich) *et simplicius et utilius est prisma habens angulum variabilem ex vitro. Videtur, sane primo aspectu impossibile ejusmodi prisma, saltem satis idoneum.* Opera, tom. i. p. 4.

mend them to subsequent observers; and we accordingly find, that one of the latest writers upon this branch of optics, has varied the refracting angle of his prisms by the mere addition of prisms of a smaller size.

The instrument employed by Clairaut, was nothing more than a plano-cylindrical lens, in which different parts of the cylindrical surface formed different angles with the plane side of the lens. In transmitting a beam of light, however, through a round aperture on the curved superficies of this plano-circular prism, different portions of the beam fall upon it at different angles of incidence; and hence arises a dispersion of the refracted rays, which confuses the prismatic spectrum. In order to remedy this evil, P. Abat, an optician at Marseilles, suggested a very elegant construction, which, with some improvements, was adopted by Boscovich. He joined together two of these plano-cylindrical lenses, one of which was plano-convex, and the other plano-concave. The concave being placed upon the convex surface, and the one being moved upon the other, the angle of the two plane surfaces was obviously varied. The disadvantages of this construction, arise from the difficulty of polishing two surfaces so as to fit each other with accuracy; from the reflections which take place at these surfaces;

from the injury done to the surfaces by their mutual friction; and from the trouble of measuring the varying angle of the refracting planes.

The instrument which we propose to substitute in the place of these contrivances, is completely free from all the objections which we have mentioned. The principle of its construction is of the most general nature, and is equally applicable to solid prisms, and to prisms consisting of a fluid included between two plates of glass.

When we view the sun through a prism, the image of that luminary is an elongated coloured spectrum, pointing in the direction of its length to the sun itself. If the prism is turned round, in a plane parallel to the plane which bisects the refracting angle, the coloured spectrum will likewise turn round the real sun, keeping always at the same distance from it, and preserving its colour and its elongated form. By this motion of rotation, therefore, the refracting angle of the prism is not varied with respect to the sun, as the refraction and dispersion are precisely the same in every part of its circular motion. From viewing the subject in this manner, the particular contrivance which we are to describe has escaped the notice of all optical writers; and it never once occurred to them, that the refracting angle of a prism may be actually varied, merely by giving it

a rotatory motion in the plane which bisects the refracting angle. *

In order to explain this seeming paradox, let AO, BO, CO, DO, EO, &c. Plate X. Fig. 9. be a number of black lines forming equal angles, AOB, BOC, &c. and let them be viewed by a prism, having the common section of its refracting planes parallel to CG, or perpendicular to AE. Then, since the refraction is made in the direction EA, the line CG will be more tinged with colour at its edges than any of the other lines, the red and yellow colour being on the right, and the blue and violet on the left of the line. The lines BF and DH will be tinged with less colour, while the line AE will appear perfectly distinct, without the least degree of colour at its edges. If the prism is now turned round in the plane which bisects the refracting angle, till the common section of the refracting planes is parallel to DH, and perpendicular to BF, the refraction will be made in the direction BF; the line DH will be coloured in the same manner as CG was coloured before; CG and AE will have

* The dispersion of any prism may also be corrected with another which produces less dispersion, by giving the latter an angular motion in a plane perpendicular to the plane which bisects the refracting angle; but this method is attended with disadvantages, which will prevent it from being put in practice.

the same colour that BF had formerly, while BF will be completely unaffected with colour. In like manner, by continuing to turn the prism in the same plane, till the common section of its refracting surfaces is successively perpendicular to CG and DH, the lines CG and DH will be altogether free of any prismatic tinge. Hence it follows, that in giving a rotatory motion to a prism in the plane which bisects its refracting angle, there is always one line in reference to which there is neither refraction nor colour, and that this line is perpendicular to another line in which the refraction and colour are a maximum. While the prism, therefore, performs one-fourth of a revolution, its refracting angle may be considered as having varied from 0° to the real angle of its refracting surfaces. Thus, in Plate X. Fig. 10. let ABCDE be a prism, ABCD one of its refracting surfaces, ADE its refracting angle, and CD the common section of its refracting planes. From any point O, draw O b parallel to CD, and O d perpendicular to O b. Then, it is obvious, that in the direc- O b, the refracting angle of the prism may be considered as 0, while, in the direction O d, the refracting angle is equal to ADE; and in any intermediate direction O c, the refracting angle will be equal to Sin. bOc × ADE, or Cos. cOd × ADE.

In applying this principle to the measurement of refractive and dispersive powers, it is necessary to have a standard prism of flint-glass, or crown-glass, the refracting angle, and the refractive and dispersive powers of which have been accurately determined. Another prism, composed of two parallel planes of glass, but with a much smaller refracting angle, must also be constructed for holding the different fluids that are to be subjected to examination. The second of these prisms must be fixed upon the ring OO, at the end of the arm G of the goniometer, Plate IV. Fig. 2. or any other circular instrument with an open centre, so as to remain stedfast in that position. The standard prism must be fixed to the ring at the centre of the circle AB, nearer the eye of the observer, and in such a manner that the two prisms may refract in opposition, and that the index E may point to zero, (the scale beginning at 90°,) when the common section of the refracting planes of the one is parallel to the common section of the refracting planes of the other. In this situation, the greatest refracting angle of the one is exactly opposed to the greatest refracting angle of the other. Let the common section of the refracting planes be in a vertical line, and let the observer view through the combined prisms, a well defined and straight vertical line AB,

Plate XI. Fig. 1. upon a light ground, like the bar of a window seen against the sky. Since the refraction of the standard prism is supposed to be greater than that of the fixed prism, the refracted image A′B′ of the object AB will appear separated from the object AB. Let the standard prism, therefore, be turned round by means of the finger screw Q, till the refracted image A′B′ comes into the position *a b*, and coincides exactly with the object AB, and let the degrees and minutes pointed out by the index be carefully marked. Then, if A be the greatest angle of the standard prism, and M the arch pointed out by the index, we shall have Sin. $M \times A$ for the refracting angle of the prism, when the direct and refracted images were coincident. Since the refracting angle, therefore, of both prisms, and the refractive power of the standard prism, are known, the refractive power of the substance included in the fixed prism may easily be found, from the formulæ to be given in this Chapter.

If the sides of the rectilineal object AB are completely free from colour when the coincidence takes place, then the standard prism has the same dispersive power as the substance in the fixed prism, provided their refractive power be the same; for, at the same angle of refraction, the dispersion of the fixed prism is exactly corrected

by that of the standard prism. If the sides of the object, however, are tinged with colour, turn round the standard prism in the same direction as before, and if the colour increases, the dispersive power of the fixed prism exceeds that of the standard one; if, on the contrary, the colour diminishes, the dispersive power of the standard prism exceeds that of the fixed one. In order to find the real dispersive power of the fixed prism, turn the standard prism till the vertical line is completely free from colour, and let m be the angle pointed out by the index. The refracting angle of the standard prism, which corrects the colour of the fixed prism, will then be Sin. $m \times A$, from which the dispersive power of the substance contained in the fixed prism may be readily deduced. In these experiments, the fixed prism must be placed in such a position with respect to the object AB, that a line joining the object and the prism may be perpendicular to the first refracting surface, which may be easily effected by the most simple contrivances. By this means, the incident rays will suffer no refraction at the first surface, and the formulæ for finding the refractive and dispersive powers of the fixed prism, will be much less complicated than if the refraction at the first surface had been introduced. The following formulæ,

which are very simple, are nearly the same with those which were given by Boscovich.

Formulæ for finding the Refractive and Dispersive Powers of the Fixed Prism.

Angle of the fixed prism . . . . A

Angle of the variable prism, (or Sin. $M \times A'$, A' being the refracting angle of the variable prism,) by which the refraction of the fixed prism is corrected . . . . a

Angle of the variable prism (or Sin. $m \times A'$) which corrects the dispersion of the fixed prism . . . . α

Index of refraction of the fixed prism . . . R

Index of refraction of the variable prism . . r

The portion of the mean refraction to which the dispersion is equal . . . . . . $d\,R$, $d\,r$

The dispersive power of the fixed prism $D = \frac{d\,R}{R-1}$

Then for the refractive power we shall have

$$\text{Sin.}\,\overline{a-x} = \frac{\text{Sin.}\,\overline{a-A}}{R}.$$

The sine of $a-x$ being found from this formula, the sine of x may be easily found. We shall then have

$$R = \frac{r \times \text{Sin. } x}{\text{Sin. A}}.$$

When the prisms do not differ much from each other in refractive power, such as prisms made of different kinds of glass, the following more simple formulæ may be used. Thus make

$$p = a - A; \text{ and } q = p - \frac{p}{r}, \text{ then}$$

$$R = r + r \text{ Sin. } q \text{ Cot. A.}$$

In order to find the dispersive power of the fixed prism, we have

$$\text{Sin. } x' = \frac{R}{r} \times \text{Sin. A};$$

and when x' is thus found, we have

$$\frac{dR}{dr} = \frac{R}{r} \times \text{Tang. } \overline{\alpha - x'} \times \text{Cot. } x' + 1,$$

$$dR = dr \times \frac{R}{r} \times \text{Tang. } \overline{\alpha - x'} \times \text{Cot. } x' + 1, \text{ and}$$

$$D = \frac{dr \times \frac{R}{r} \times \text{Tang. } \overline{\alpha - x'} \times \text{Cot. } x' + 1}{R - 1}$$

R being the index of refraction for the mean refrangible rays, dR is a part of the whole refraction, and is always equal to the difference between the index of refraction for the first red ray, and the index of refraction for the last violet ray.

In measuring the dispersive powers of different kinds of flint or crown glass, by means of a standard prism of the same glass, or in every case where R is nearly equal to r, we have $x'=A$; and therefore the formula may be greatly simplified, and will then become

$$D=\frac{dr \times \text{Tang. } a-A \times \text{Cot. } A+1}{R-1}.$$

If it should happen, that the fixed prism contained a substance of such a high refractive or dispersive power, that the greatest angle of the standard prism was incapable of correcting its refraction and dispersion, the standard prism may then be fixed in the place of the other, while the other prism has its refracting angle reduced by a rotatory motion, till it corrects the refraction and dispersion of the standard prism.

As the maximum angle of both the prisms may be determined with the utmost accuracy, and as the scale * for measuring the variation of the refracting angle of the standard prism is sufficiently large to ascertain the most minute changes, the accuracy of the results must depend princi-

* The magnitude of the scale evidently increases as the maximum angle of the prism diminishes. If the maximum

pally upon the precision with which the coincidence of the images, and the complete correction of colour, are observed. In order to increase the accuracy of these observations, the goniometer containing the two prisms might be placed before the object-glass of a small telescope, through which the observer must examine the object AB, and its fringes of colour. By this means the fringes will be greatly magnified, and the position of the standard prism when the refracted image of AB is perfectly achromatic, will afford a measure of the angle to which it has been reduced. If there should be any incorrected colour in the telescope itself, it may be easily distinguished from the colour produced by the prisms; but as the aperture of the object-glass must be necessarily very small, and as the magnifying power does not require to be great, this incorrected colour can never be of any consequence.

Dr Blair, in his ingenious paper on the *Unequal Refrangibility of Light*, has maintained, after Clairaut and Boscovich, that the proportion of the coloured spaces varies with the diffe-

angle is 20°, we have an arch of 90° as a scale to measure the variation of the refracting angle from 0° to 20°; whereas, if the maximum angle is only 5°, we have an arch of 90° to measure the variation from 0° to 5°.

rent substances that are employed, and therefore a complete correction of colour cannot be effected by two media of different dispersive powers. This singular fact has been controverted by Dr Wollaston, who found, in all the substances which he tried, that the coloured spaces had the same proportion to each other in similar positions of the prism.* Dr Blair, we have rea-

* When Boscovich observed, after Clairaut, the irrationality of the coloured spaces in the prismatic spectrum formed by different substances, he was very unwilling to acquiesce in such a singular result, without the most satisfactory evidence. Suspecting that there was some source of error in his experiments, he repeated them with the greatest caution, and calculated the magnitude of the errors which might arise from the particular circumstances of the experiment. He at last admitted the irrationality of the coloured spaces as a fact demonstrated by incontrovertible experiments, and has shewn how three of the colours in the spectrum may be corrected in achromatic telescopes. The same opinion was admitted, upon the evidence of experiment, by our celebrated countryman, the late Dr John Robison. To those, however, who may think that Dr Wollaston's opinion is founded on more direct evidence, we propose the following experiment as decisive of the question. Take a prism of oil of cassia, and another of crown glass, and vary the angle of the one which produces the greatest degree of colour, till the transmitted light is as achromatic as possible. In this position, the quantity of uncorrected colour is so very great, that it cannot be ascribed to any other cause than to an inequality in the corresponding spaces of the spectra formed by the crown glass and the oil of cassia. See Book V. Chap. I. where this subject will be treated at considerable length.

son to know, ascribes the results obtained by Dr Wollaston to his not having used lenses, in which the uncorrected colour, by being greatly magnified, is rendered more apparent than when prisms alone are used; but in the course of my experiments upon dispersive powers, I have observed the uncorrected colour in the form of green and wine-coloured fringes, even with prisms; and I have perceived them in many substances which were not examined either by Boscovich or Dr Blair. When the dispersive power, therefore, of substances in which this irrationality exists, is measured by the preceding method, the appearance of the green and wine-coloured fringes will mark the position of the standard prism, which corrects the dispersion of the substance under examination.

The subject of dispersive powers has, till within these few years, been investigated merely for the purpose of discovering achromatic combinations for the improvement of the telescope. The dispersions of two or three different kinds of glass, and of a few fluids, were numerically ascertained; but no attempt was made to consider the subject in a general manner, or to investigate it as a separate branch of science, exhibiting the most curious results, and unfolding new properties of

transparent bodies. Dr Wollaston had the honour of beginning this important inquiry: and he determined the order of dispersive powers for 33 substances, without, however, giving any numerical estimate of their magnitude. By means of the instrument which has been briefly described, I have ascertained, in numbers, the dispersive powers of more than a hundred transparent substances, the greater part of which were never before examined; and I have obtained many results of a most unexpected and singular kind. But before I proceed to explain these results, it will be necessary to give a more particular account of the method by which they were obtained.

The instrument which was employed in these experiments, is represented in section in Fig. 2. of Plate XI. The circular head AB, similar to that which appears in Fig. 1. Plate VI. is divided into 360 degrees; and has a tubular shoulder, *e*, *e*, which moves upon the tube $d\,d'\,d\,d'$. Upon the tube $d\,d'$, $d\,d'$ is fixed another small tube, that carries the arm $d\,c$, on the circumference of which is the vernier scale for subdividing the degrees on the circular head. The extremity $d\,d$ of the tube $d\,d'$, $d\,d'$ is fixed into the stand CD, and upon its other extremity $d'\,d'$, which terminates in a ring, is fixed the prism *m*, whose dispersive

power is required. The standard prism *n*, having such a refracting angle as to produce a greater dispersion than that of *m*, is fastened to a tube *fg*, shewn separately in Fig. 3. which screws upon the outer surface of the shoulder *e e'*. It is obvious, from the preceding description, that when the circular head AB is turned round, the tube *fg*, with the standard prism *n*, will be carried along with it, while the vernier *c* and the other prism *m* remain stationary; and if the inner surfaces of the two prisms were parallel to each other, and perpendicular to the axis of motion, in any given position of the circular head, this parallelism will be preserved in every other position.

A horizontal bar, AB * Fig. 4. about three or four inches broad, and having its sides perfectly straight and parallel, is stretched across the window, so as to be exactly perpendicular to a plumb line CD suspended from the top of the window. The instrument is then placed at a convenient distance from the bar, and in such a position, that a line joining the eye of the observer at O, Fig. 2. and the centre of the bar at E, Fig. 4.

* If AB were a rectangular luminous space, placed on a dark ground, it would answer the purpose equally well.

may be perpendicular to the anterior surface of the prism *m*, and to the bar AB. When this adjustment is made, the whole of the instrument is turned round in the tube which forms the top of the stand, till the common section of the refracting surfaces of the prism *m* is perpendicular to CD *, and it is fixed in this position by the screw at *s*. If the bar AB be now viewed through the prism *m*, the lower side of it will be bordered with red and yellow, and the upper side with blue and violet; but after the tube *fg*, with the standard prism *n*, is screwed upon the shoulder *e e′*, so as to refract in opposition to *m*, with the common section of its planes turned upwards, and perpendicular to CD, the red and yellow fringes will appear on the upper side of AB, while the blue and violet fringes occupy its lower side, on account of the superior dispersion produced by the standard prism. In order to diminish the refracting angle of the prism *n*, turn round the circular head AB towards the right hand, the eye continuing to observe the image of the bar AB Fig. 4. and the coloured fringes will gradually diminish. As soon as the fringes vanish, and the

* This adjustment may be easily made, by observing when the image of CD, seen hrough the prism, is coincident with CD, when seen directly by the other half of the pupil.

bars appear completely free from colour, mark the degree and minute of the circular head which is indicated by the vernier. Turn the circular head towards the left hand, till the fringes again disappear, and mark the degree and minute pointed out by the index. Then if φ be the arch comprehended between these two positions, and B the angle of the standard prism n, the refracting angle α, to which this prism has been reduced in order to correct the dispersion of the prism m, will be obtained from the following formula, viz.

$$\alpha = \text{Cos.} \frac{\varphi}{2} \times \text{B.}*$$

Thus if the prism m has an angle of 24° 39′, and contains water, and if n is a prism of flint glass, we shall have $\varphi = 156°$ when B is equal to 41° 11′, and consequently $\alpha = \text{Cos. } 78° \times 41° \, 11' = 8° \, 34'$, that is, a prism of flint glass of 8° 34′ corrects the dispersion of a prism of water of 24° 39′. Hence, if the refractive power of water is known, and also the refractive and dispersive powers of flint glass, the dispersive power of water may be easily determined from the formula in page 299. As an example of this calculation, we shall take

* In order to save the trouble of dividing the arch by 2, the divisions on the limb might be numbered only to 180 instead of 360°.

the case of water and flint glass, which has been already mentioned.

Angle of the prism of water -	$24° 39' = A$
Angle of the prism of flint glass	$41° 11' = B$
Reduced angle of the prism of flint glass, which corrects the dispersion of the prism of water	$8° 34' = \alpha$
Refractive power of the prism of flint glass - - -	$1.616 = r$
Refractive power of water -	$1.336 = R$
The portion of the mean refraction of flint glass to which the dispersion is equal - -	$0.0320 = dr$

$R = 1.336$. Log. 0.125806

$r = 1.616$. Log. 0.208441

$\frac{R}{r}$ - - 9.917365

Sin. $A = 24° 39'$ 9.620213

Sin. $x' = 20° 10'$ 9.537578

$\alpha = 8° 34'$

$x' = 20° 10'$

$-\overline{\alpha - x'}$ $11° 36'$

Cotangent of x'	10.435017
Tangent of $\alpha - x'$	9.312327
-0.5589	9.747344
$+1.$	
.4411	9.644537
$\times\frac{}{r}$	9.917365
$\times dr$	8.505150
$dR = 0.0117$	8.067052
$R-1 = .336$	9.526339
$\frac{dR}{R-1} = 0.0347$	8.540713

Hence it appears, that the dispersive power of water is 0.0347.*

When a fluid is included in the prism *m*, it must necessarily be confined between two plates of glass, and therefore there are no fewer than six refractions before the incident rays reach the eye of the observer. The indistinctness and loss of light which is, to a certain extent, occasioned by the refractions and reflections that take place at

* Another observation upon water, with a different prism of flint glass, makes $dR = 0.0119$, and $\frac{dR}{R-1} = 0.0352$.

these different surfaces, may in a great measure be removed by the contrivance represented in Fig. 5. Plate XI. where AB is the circular head, *m* the fixed prism, and *n o* a piece of parallel glass, of a rectangular form, which is moveable round *r* in a frame at the extremity of the bent arm *r st*. The lengths *r n*, *r o* are unequal; so that when the piece of glass is turned round to make the end *o* fall upon the prism *m*, it may form a different angle with the surface. The fluid is in some cases preserved between the prism *m*, and the plate of glass *n o*, by capillary attraction, and it keeps its position even when the arm *r s t*, and the piece of glass *n o*, are moved round along with the circular head AB. By this means we get rid of two refractions, and of any small error that might arise from a want of parallelism between the interior surfaces of the prism. There are some inconveniences, however, in this contrivance, which have prevented me from making frequent use of it.

By these methods, the dispersive powers of all the substances contained in the following Table, have been carefully measured and computed. The first column contains the values of $\frac{d\,R}{R-1}$, which is the natural measure of the dispersive power; and the second column contains the values of $d\,R$, or

the part of the whole refraction to which the dispersion is equal.

The different dispersive powers comprehended in the Table, vary from 0.022, the dispersive power of cryolite, to 0.400, the estimated dispersive power of the greatest refraction of chromate of lead;—an interval of surprising magnitude, and particularly interesting, when we recollect that both Newton and Euler considered all transparent bodies as possessing the same powers of dispersion.

The substances at the head of the Table, between the dispersive powers of 0.0128 and 0.400, have never before been the subject of experiment, and present us with results of unexpected magnitude. Chromate of lead, realgar, and phosphorus, which are included within these limits, might, from their chemical properties, be supposed to possess a great degree of dispersion; but the oil of cassia, which exceeds even phosphorus in dispersive power, and stands far above every animal or vegetable product, exerts a most surprising power in separating the extreme rays, and indicates the existence of some ingredient which chemical analysis has not been able to detect.

In comparing the refractive and dispersive powers of transparent bodies, it is difficult to dis-

cover any general principle, upon which these properties depend.

In the two simple inflammable substances, sulphur and phosphorus, and in the metallic salts, a high refractive density is accompanied with a high power of dispersion.

In the precious stones, on the contrary, a great refractive power, exceeding that of flint glass, is attended with a dispersive power generally much lower than water.

The dispersive powers of the resins, gums, oils, and balsams, greatly exceed that of water, and correspond in some measure with their powers of refraction.

The different kinds of glass coloured with metals have a higher dispersive as well as a higher refractive power than flint glass.

The muriatic, the nitric, and the nitrous acids, have dispersive powers considerably above water; while the sulphuric, the phosphoric, the citric, and the tartaric acids, which surpass the former in refractive density, possess very inferior powers of dispersion.

Fluor spar and cryolite, the only minerals in which fluoric acid is a principal ingredient, have the lowest dispersive powers of all bodies, and

the lowest refractive powers of all solid substances.*

The most singular result, however, which is contained in the following Table, relates to the dispersive powers of doubly refracting substances. The first experiment which I made upon these crystals, was to determine the dispersive power of Iceland spar, and from a cause merely accidental, I corrected the colour of the least refraction. The result thus obtained was 0.026, considerably below water, which stands at 0.035 of the scale; and upon comparing it with the place assigned to Iceland crystal by Dr Wollaston, I was surprised to find that he placed its dispersive power very considerably above water, and even above diamond. This unexpected difference between the two measures, induced me to repeat the experiments, not only with other prisms of the Iceland spar, but also with other standard prisms of flint and crown glass. These new results served only to confirm the accuracy of the first experiment, and to strengthen my suspicion that Dr Wollaston had committed some mistake. As this reasoning, however, was founded on the assumption

* The topaz, which contains from 17 to 20 parts of fluoric acid, has a dispersive power nearly as low as fluor spar; but, like the other precious stones, it has a great power of refraction.

which both Dr Wollaston and I had made, that the spar had only one dispersive power, I resolved to measure the dispersive power of the extraordinary refraction. This new value having turned out to be greater than that of water, I immediately saw that Dr Wollaston had measured the colour of the greatest refraction, while I had measured the colour of the least; and that this remarkable mineral, which had so long perplexed philosophers by its double refraction, possessed the no less extraordinary and inexplicable property of two dispersive powers. In subjecting to examination other crystals that afforded double images, such as carbonate of strontites, carbonate of lead, and chromate of lead, I found that every separate refraction possessed a separate dispersive power. This general law, though not repugnant to any optical phenomena, is still of such a nature, that it could not have been inferred *a priori* from any relation which is known to subsist between the refractive and dispersive powers. No person, indeed, has even conjectured, that a double dispersive should accompany a double refractive power;* and if we were to reason in this

* In a Table of refracting powers, published by Mr Cavallo in his Elements of Natural Philosophy, he has added, from different authors, a numerical estimate of the dispersion, or dissipation, as he calls it, of a very few substances, and for each

case from an analogy founded on experiment, an analogy, too, which is by no means remote, we should certainly conclude, contrary to the fact, that the greatest refractive power would be accompanied with the least power of dispersion. In all the minerals in which a metal is the principal ingredient, those which have the greatest refractive density, have also the greatest faculty of producing colour; while in all the precious stones a high refractive power is attended with a low power of dispersion. This remarkable property of a double dispersion, therefore, is contrary to the general results indicated by experiment; and though it appears to exclude some of the theories by which a double refraction has been explained, it certainly adds another to those numerous difficulties with which philosophy has yet to struggle, before she can reduce to a satisfactory generalisation the anomalous and capricious phenomena which light exhibits in its passage through transparent bodies.

refraction of the Iceland spar, he has given a separate measure of its *dispersion*. These, however, are not measures of its two dispersive powers, as will be perfectly obvious by inspecting the Table, but merely of the quantity of colour produced by each refraction, which is of course proportioned to the refractions themselves;—in the same manner as two prisms of flint glass, with different angles, have two *dispersions*, though the *dispersive power* of both is the same.

*Table of the Dispersive Powers of various Substances.**

Names of the substances.	Dispersive power or values of $\frac{dR}{R-1}$	Part of the whole refraction to which the dispersion is equal, or values of dR.
†Chromate of lead (greatest refraction) estimated at	0.400	0.770
Chromate of lead (greatest refraction) must exceed	0.296	0.570
Realgar, a different kind, melted .	0.267	0.394
‡Chromate of lead (least refraction)	0.262	0.388
Realgar, melted	0.255	0.374
Oil of cassia	0.139	0.089
*Sulphur after fusion . . .	0.130	0.149
Phosphorus	0.128	0.156
*Balsam of Tolu	0.103	0.065

* The substances in the Table marked with an asterisk, are those which Dr Wollaston has examined in determining the *order* of their dispersive powers.

† This estimated value of the dispersive power of the second refraction of chromate of lead, is founded on the following observation. A prism of oil of cassia, whose refracting angle is 59° 30′, does not nearly correct the dispersion of the greatest refraction of a prism of chromate of lead, whose refracting angle is 9° 16′. The uncorrected colour is not much inferior to the whole colour produced by the least refraction.

‡ This value is obtained from the following observation: A prism of oil of cassia, with a refracting angle of 39° 15′, corrects the dispersion of the least refraction of a prism of chromate of lead, having a refracting angle of 9° 16′.

Balsam of Peru	0.093	0.058
Carbonate of lead (greatest refraction)	+0.091	+0.091
Barbadoes aloes	0.085	0.058
Oil of anise seeds . . .	0.077	0.044
Balsam of styrax . . .	0.067	0.039
*Guiacum	0.066	0.041
Carbonate of lead, (least refraction)	0.066	0.056
Oil of cummin	0.065	0.033
Gum Ammoniac . . .	0.063	0.037
Oil of Barbadoes tar . . .	0.062	0.032
Oil of cloves	0.062	0.033
Green coloured glass . . .	0.061	0.037
Sulphate of lead	0.060	0.056
Deep red glass	0.060	0.044
*Oil of sassafras . . .	0.060	0.032
Opal coloured glass . . .	0.060	0.038
Rosin	0.057	0.032
Oil of sweet fennel seeds . .	0.055	0.028
Oil of spearmint . . .	0.054	0.026
Orange coloured glass . .	0.053	0.042
Rock salt	0.053	0.029
Caoutchouc	0.052	0.028
Oil of Pimento	0.052	0.026
*Flint glass	0.052	0.032
Deep purple glass . . .	0.051	0.031
Oil of angelica . . .	0.051	0.025
Oil of thyme	0.050	0.024
Oil of feugreck . . .	0.050	0.024
Oil of wormwood . . .	0.049	0.023
Oil of pennyroyal . . .	0.049	0.024
Oil of caraway seeds . . .	0.049	0.024

Oil of dill seeds	0.049	0.023
Oil of bergamot	0.049	0.023
†Flint glass	0.048	0.029
Chio turpentine	0.048	0.028
Gum thus	0.048	0.028
Oil of lemon	0.048	0.023
Flint glass	0.048	0.028
Oil of juniper	0.047	0.022
Oil of chamomile . . .	0.046	0.021
Gum juniper	0.046	0.025
Carbonate of strontites (greatest refraction)	0.046	0.032
Oil of brick	0.046	0.021
Nitric acid	0.045	0.019
Oil of lavender . . .	0.045	0.021
Balsam of sulphur . . .	0.045	0.023
Tortoise shell	0.045	0.027
Horn	0.045	0.025
*Canada balsam . . .	0.045	0.024
Oil of marjoram	0.045	0.022
Gum olibanum	0.045	0.024
Nitrous acid	0.044	0.018
Cajeput oil	0.044	0.021
Oil of hyssop	0.044	0.022
Oil of rhodium	0.044	0.022
Pink coloured glass . . .	0.044	0.025

† The dispersive power of the different kinds of flint glass tried by Boscovich, varies from 0.0457 to 0.0525. Dr Robison informs us, that he examined, with great care, a parcel of flint glass, whose dispersive power was 0.038. We suspect that he has committed some mistake in the reduction of his experiments.

Oil of savine	0.044	0.021
Oil of poppy	0.044	0.020
*Jargon, (greatest refraction) .	0.044	0.045
Muriatic acid . . .	0.043	0.016
*Gum copal	0.043	0.024
Nut oil	0.043	0.022
Burgundy pitch . . .	0.043	0.024
*Oil of turpentine . . .	0.042	0.020
Oil of rosemary	0.042	0.020
Feldspar	0.042	0.022
Glue	0.041	0.022
*Balsam of Capivi . . .	0.041	0.021
*Amber	0.041	0.023
Oil of nutmeg	0.041	0.021
Stilbite	0.041	0.021
Oil of peppermint . . .	0.040	0.019
Spinelle ruby	0.040	0.031
*Calcareous spar, (greatest refraction)	0.040	0.027
Oil of rapeseed	0.040	0.019
Bottle glass	0.040	0.023
Gum Elemi	0.039	0.021
Sulphate of iron . . .	0.039	0.019
*Diamond	0.038	0.056
Oil of olives	0.038	0.018
Gum mastich . . . :	0.038	0.022
White of an egg . . .	0.037	0.013
Oil of rhue	0.037	0.016
Gum myrrh	0.037	0.020
Beryl	0.037	0.022
Obsidian	0.037	0.016

Ether	0.037	0.012
*Selenite	0.037	0.020
*Alum	0.036	0.017
Castor oil	0.036	0.018
Sulphate of copper	0.036	0.019
§*Crown-glass, very green	0.036	0.020
Gum Arabic	0.036	0.018
Sugar after being melted and cooled	0.036	0.020
Jelly fish, body of, (*Medusa Æquorea*)	0.035	0.013
Water	0.035	0.012
Aqueous humour of a haddock's eye	0.035	0.012
†Vitreous humour of do.	0.035	0.012
Citric acid	0.035	0.019
Rubellite	0.035	0.027
Leucite	0.035	0.018
Epidote	0.035	0.024
Garnet	0.033	0.027
Pyrope	0.033	0.026

§ The dispersive power of the different kinds of common glass (strass), tried by Boscovich, varies from 0.0330 to 0.0346. Dr Robison found the dispersive power of some crown glass made at Leith, so low as 0.027.

† I was very anxious to ascertain the dispersive power of the exterior and interior part of the crystalline; but I found this quite impracticable, from the impossibility of finding any portion of it of an uniform refractive power. Owing to the gradual increase of its refractive density towards the centre, a distinct image could not be perceived through any part of it; and upon attempting to convert into a prism the whole of the lens, which was about .32 of an inch in diameter, I found that, though confined between two parallel planes of glass, it had a focal length of .85 of an inch, and therefore could not be employed for this purpose.

Chrysolite	0.033	0.022
Crown glass	0.033	0.018
Oil of ambergrease . . .	0.032	0.012
Oil of wine	0.032	0.012
Phosphoric acid, solid prism .	0.032	0.017
*Plate glass	0.032	0.017
*Sulphuric acid	0.031	0.014
Tartaric acid	0.030	0.016
Borax	0.030	0.014
Axinite -	0.030	0.022
*Alcohol	0.029	0.011
*Sulphate of barytes . . .	0.029	0.019
Tourmaline	0.028	0.019
Carbonate of strontites, (least refraction)	0.027	0.015
*Rock crystal	0.026	0.014
Emerald	0.026	0.015
Calcareous spar, (least refraction) .	0.026	0.016
Blue sapphire	0.026	0.021
Bluish topaz from Cairngorm .	0.025	0.016
Chrysoberyl	0.025	0.019
Blue topaz, from Aberdeenshire, .	0.024	0.025
Sulphate of strontites . . .	0.024	0.015
*Fluor spar	0.022	0.010
Cryolite	0.022	0.007

In the course of my experiments on refractive and dispersive powers, I have been under great obligations to Sir George Mackenzie, Bart. Professor Jameson, and Thomas Allan, Esq. who furnished me with many minerals which I could not otherwise have obtained.

The dispersive powers of transparent bodies might be deduced from another method of observation, which, though not equal in accuracy to that which has been described, may, under particular circumstances, be deserving of attention.

When we look through a prism at a horizontal bar, it is tinged on one side with red and yellow, and on the other side with indigo and violet. If the angle subtended by the bar is considerable, a dark space will intervene between the violet and the red; but if the angle subtended by the bar is diminished by viewing it at a greater distance, this dark interval likewise diminishes, and as soon as it vanishes, the visible extremity of the red comes in contact with the visible extremity of the violet. The contact of these colours is indicated by the *commencement of a pink fringe.* The distance therefore between the bar and the eye of the observer, will afford a measure of the dispersive power, when the angle of the prism and its refractive power are determined. If it should be thought convenient to make the observations at a constant distance from the bar, the angle of the prism may be varied by the method already described, till it produces the pink fringe at the union of the red and violet.

CHAP IV.

On the New Properties impressed upon Light by its transmission through Diaphanous Media, and by its reflection from the polished Surfaces of opaque and transparent Bodies.

ALTHOUGH the subject of this Chapter does not involve the description of any instrument, and is therefore not strictly connected with the present work, yet, as it forms a branch of the same enquiry with the subject of refractive and dispersive powers, and particularly with that of double refraction, I hope it will not be considered either as an unimportant or uninteresting digression.

There is, perhaps, no subject within the whole range of physical science, which presents such singular and capricious results as that of double refraction; and there is certainly none upon which the genius and industry of philosophers have been so fruitlessly expended. From the time of Bartholinus and Huygens, who first observed the action of Iceland spar upon light, to the beginning

of the 19th century, it has exercised the abilities of the most eminent mathematicians, and yet during this vast interval it has scarcely been enriched with a single fact; no explanation, that could be listened to for a moment, has ever been proposed, nor has it received the smallest aid from collateral enquiries. Even Newton and Laplace have retired without laurels from the arduous research; and had not a discovery of a most unexpected nature given a new form and character to the investigation, it might have long remained among the impenetrable secrets of Nature.

That the experiments and reasonings which are to occupy this Chapter, may be readily understood by those who are not familiar with the subject, I shall endeavour to give a short and perspicuous view of the phenomena of double refraction, and of the new modification of light which has been recently discovered by Malus.

If a ray of light fall upon one of the surfaces of a rhomboid of Iceland crystal, or calcareous spar, and is transmitted through the opposite surface, it is separated into two pencils, one of which proceeds in the direction of the incident ray, while the other forms with it an angle of 6° 16′. The first of these pencils is said to experience the *usual*

or *ordinary* refraction, and the other the *unusual* or *extraordinary* refraction. Hence, if the luminous object from which the ray of light proceeds is looked at through the crystal, two images of it will be distinctly visible, and will continue so even when the rhomboid is turned round the axis of vision. If another rhomboid of Iceland spar is placed behind the first, in a similar position, the pencil which suffered the ordinary refraction from the first rhomboid will suffer the same refraction by the second, and the pencil that experienced the extraordinary refraction from the first will experience only the extraordinary refraction from the second,—none of the pencils being separated into two as before. . But if the second rhomboid is turned slowly round, while the first remains stationary, each of the pencils begins to separate into two; and when the eighth part of a revolution is completed, the whole of each of the pencils is divided into two portions. When the fourth part of a revolution is finished, the pencil refracted in the ordinary way by the first crystal will be refracted in the extraordinary way only by the second, and the pencil refracted in the extraordinary way by the first, will be refracted in the ordinary way only by the second, so that the four pencils will be again reduced to two. At the end of $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ of a revolution, the same pheno-

mena will be exhibited as at the end of $\frac{1}{8}$ part of a revolution. At the end of $\frac{1}{2}$ of a revolution, the same phenomena will be seen as at the first position of the crystals; and at the end of $\frac{3}{4}$ of a revolution, the same phenomena will be exhibited as at the end of $\frac{1}{4}$ of a revolution.

If we now look at a luminous object through the two rhomboids, we shall, at the commencement of the revolution, see only two images, viz. one of the least and one of the greatest refracted images. At the end of $\frac{1}{8}$ of a revolution, four images will be seen. At the end of $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of a revolution, the images will be again reduced to two; and at the end of $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$ of a revolution, four images will be visible, every image having vanished and reappeared in its turn.

It is obvious, therefore, that the light which forms these images, has suffered some new modification, or acquired some new property, which prevented it, in particular parts of a revolution, from penetrating the second rhomboid. This property has been called *polarisation*; and light is said to be *polarised* by passing through a rhomboid of calcareous spar, or any other doubly refracting crystal.

Almost all crystallised substances possess the property of double refraction, and consequently

the power of *polarising* light. The most important of these, arranged in the order of their refractive powers, are the following:

Chromate of lead.	Topaz.
Carbonate of lead.	Tartaric acid.
Jargon.	Rock crystal.
Epidote.	Sulphate of copper.
Carbonate of Strontites.	Selenite.
Chrysolite.	Sulphate of iron.
Calcareous spar.	

These different crystals, and many others, exhibit all the phenomena which we have already described as produced by Iceland spar.

A few years ago, M. Malus, a colonel of engineers in the French army, announced the discovery of a new property of *reflected light*. He found, that when light is reflected at a particular angle from all transparent bodies, whether solid or fluid, it had acquired, by reflection, that remarkable property of *polarisation*, which had hitherto been regarded as the effect only of double refraction.

If the light of a taper, reflected from the surface of water at an angle of 52° 45′, is viewed through a rhomboid of Iceland crystal, which can be turned about the axis of vision, two images of the taper will be distinctly visible in one position of the crystal. At the end of $\frac{1}{8}$ of a revolution, one

of the images will vanish; and it will re-appear at the end of $\frac{1}{4}$ of a revolution. The other image will vanish at the end of $\frac{3}{8}$ of a revolution, and will re-appear at the end of $\frac{1}{2}$ a revolution; and the same phenomena will be repeated in the other two quadrants of its circular motion. The light reflected from the water, therefore, has evidently been *polarised*, or has received the same character as if it had been transmitted through a doubly refracting crystal.

The angle of incidence at which this modification is superinduced upon reflected light, increases in general with the refractive power of the transparent body; and when the angle of incidence is greater or less than this particular angle, the light suffers only a partial modification, in the same manner as when the two rhomboids of Iceland spar are not placed either in a similar or in a transverse position.

In examining the light reflected from opaque bodies, such as black marble, ebony, &c. Malus discovered, that they also possessed the power of polarisation; and he found that polished metals had not the property of impressing this character upon light, though they did not alter it when it had been acquired from another substance.

When a ray of light was divided into two

pencils by a rhomboid of Iceland spar, Malus made these pencils fall on the surface of water at an angle of 52° 45′. When the principal section of the rhomboid (or the plane which bisects the obtuse angles,) was parallel to the plane of reflection, the ordinary pencil was partly reflected and partly refracted like any other light; but the extraordinary ray penetrated the water entire, and not one of its particles escaped refraction. On the contrary, when the principal section of the crystal was perpendicular to the plane of reflection, the extraordinary ray was partly refracted and reflected, while the ordinary ray was refracted entire. These brilliant discoveries, which throw a new light upon the phenomena of refraction and reflection, were honoured with the Rumford medal by the Royal Society of London; but unfortunately for science, Malus scarcely survived the adjudication of this high reward.

Having thus given a very brief and general view of the phenomena of light when polarised by doubly refracting crystals, and by reflection from transparent bodies, I shall now proceed to give an account of the results which I have obtained in the same field of enquiry.

1. *On a New Property of Refracted Light.*

After repeating the experiments of Malus, and measuring several of the angles of incidence at which light was polarised by reflection from different substances, I made a variety of experiments, with the view of discovering if a similar character could be impressed upon light by its transmission through bodies either wholly or imperfectly transparent. All these trials, however, afforded no new result; and every hope of discovering such a property was extinguished, when my attention was directed to a singular appearance of colour in a thin plate of agate. This plate, bounded by parallel faces, is about the 15th of an inch thick, and is cut in a plane perpendicular to the laminæ of which it is composed. The agate is very transparent, and gives a distinct image of any luminous object; but on each side of this image is one highly coloured, forming with it an angle of about 10°, and so deeply affected with the prismatic colours, that no prism of agate, with the largest refracting angle, could produce an equivalent dispersion. Upon examining this coloured image with a prism of Iceland spar, I was astonished to find that it had acquired the

same property as if it had been transmitted through a doubly refracting crystal, and upon turning the Iceland spar about its axis, the images alternately vanished at every quarter of a revolution. My attention was now directed to the common colourless image, formed by pencils transmitted perpendicularly through the agate; and by viewing it with a prism of Iceland spar, it exhibited all the characters of one of the pencils produced by double refraction, the images alternately vanishing in every quadrant of their circular motion.

When the image of a taper, reflected from water at an angle of 52° 45′, so as to acquire the property discovered by Malus, is viewed through the plate of agate, having its laminæ parallel to the plane of reflection, it appears perfectly distinct; but when the agate is turned round, so that its laminæ are perpendicular to the plane of reflection, the light which forms the image of the taper *suffers total reflection, and not one ray of it penetrates the agate.*

If a ray of light, incident upon one plate of agate, is received after transmission upon another plate of the same substance, having its laminæ parallel to those of the former, the light will find an easy passage through the second plate; but if

the second plate has its laminæ perpendicular to those of the first, the light will be *wholly reflected, and the luminous object will cease to be visible.*

Owing to a cause which will afterwards be noticed, there is a faint nebulous light, unconnected with the image, though always accompanying it, lying in a direction parallel to the laminæ. This unformed light never vanishes along with the images; and in one of the specimens of agate it is distinctly incurvated, having the same radius of curvature with the adjacent laminæ. This unformed light is represented in Plate XI. Fig. 6. where AB, CD are the directions of the laminæ, E the visible image of the candle, and F the unformed and incurvated light, in the middle of which the other image vanishes. The nebulous light surrounding the image E has now disappeared; but, by turning round the spar, it reappears by degrees, while the reviving image at F gradually dispels the haze in which it was enveloped.

This remarkable property of the agate, I have found in the kindred substances of *cornelian* and *chalcedony;* and it is exhibited in its full effect, even when these bodies are formed into prisms, and when the incident rays fall with any angle of obliquity. In one specimen of agate, which

has no veins to indicate the direction in which it was cut, the images did not vanish as before; and in another specimen, of a similar character, the images suffered only an alternate diminution of brightness, in the same manner as a pencil of light suffers only a partial modification when reflected from water, at a greater or less angle than 52° 45′.

Although the preceding results are by no means ripe for generalisation, yet I cannot omit the present opportunity of hazarding a few conjectures respecting the cause of this singular property of the agate.

May not the structure of this mineral be in a state of approach to that particular kind of crystallisation which affords double images? and, may not the unformed nebulous light be an imperfect image, arising from that imperfection of structure? We have already seen, that when the image vanishes at F, the nebulous light in the same place is a maximum, while this light gradually diminishes during the reappearance of the image. When the image, which had disappeared at F, regains its full lustre, the nebulosity with which it is encircled is very small; and this remaining light is, in all probability, no portion of the unformed image, but only a few scattered rays arising

from the imperfect transparency of the mineral. If this explanation should be rendered improbable, by observing that the small portion of unextinguished light at E is not diminished when a thinner or a purer piece of agate is employed, may we not suppose, what is most likely to happen, that the plate of agate has not been cut exactly in a plane perpendicular to the laminæ, and that a small portion of the unformed image remains, in the same manner as one of the images is not completely extinguished, when we view with a prism of Iceland crystal the light reflected from water, at an angle a little above or a little below 52° 45′.

By forming the agate into a prism, the nebulous light should be separated from the adjacent image, in proportion to the angle formed by the refracting planes; but owing, perhaps, to the smallness of its double refraction, if it has such a property, I have not observed any separation of this kind in the prisms which I have tried.

The incurvated form of the nebulous light, corresponding with the curvature of the laminæ, seems to connect it with the laminated structure of the agate, and to indicate, that the phenomena of double refraction are produced by an alternation of laminæ of two separate refractive and dispersive

powers. In calcareous spar, one set of the laminæ may be formed by a combination of oxygen and calcium, while the other set is formed by a combination of oxygen and carbon. In chromate of lead, the chromium and oxygen may give one image, while the oxygen and lead give another. In like manner the carbonate of lead, the carbonate of strontites, jargon, and other doubly refracting crystals, may afford double images, in virtue of similar binary combinations. Of the simple inflammable substances, sulphur is the only one which has the faculty of double refraction; but it will probably be found, that it holds a metal in its composition, or some other ingredient which chemical analysis has not been able to discover.

If the explanation which has now been given of the polarising power of the agate should be confirmed by future experiments, it will be considered as a case, though a very curious one, of double refraction; but if these conjectures should be overturned by subsequent observations, the phenomena which we have described must be ranked among the most singular appearances which light exhibits in its passage through diaphanous media.

2. *On the Power of Transparent Bodies to deprive Light of this new Property.*

Having thus determined that light received a new property by refraction through the agate, I was anxious to ascertain if light thus polarised suffered any change by passing through other transparent bodies; and, in the course of these researches, I have been led to results of the most unexpected and surprising nature.

When polarised light was transmitted through rock crystal, it was *depolarised,* or converted into common light, in one position; while, in another position of the crystal, the polarity of the light was undisturbed. The other substances which possessed the faculty of *depolarisation,* in one position only, were *Topaz, Chrysolite, Borax, Sulphate of Lead, Feldspar, Selenite, Citric Acid, Sulphate of Potash, Carbonate of Lead, Leucite, Tourmaline, Epidote, Mica, Iceland Spar, Agate without the polarising property,* and some pieces of *Plate Glass.*

When *polarised* light was transmitted through *gum Arabic,* it was depolarised in every position of the gum; and I found the same remarkable property in *Horn, Glue,* and *Tortoise Shell.*

The substances which had not the property of

depolarisation, and which produced nò change upon polarised light, were *Muriate of Ammonia*, *Alum*, *Amber*, *Gum Juniper*, *Rock Salt*, *Gum Mastich*, *Obsidian*, and *Diamond*; but it is probable that these bodies were not cut in the proper direction.

3. *On New Optical Properties exhibited by Mica and Topaz.*

In making the experiments with *Mica* and *Topaz*, of which I was kindly furnished with excellent specimens by Thomas Allan, Esq. I observed some singular phenomena, which seem to be peculiar to these minerals, and which I have endeavoured to represent in Fig. 7. of Plate XI.

The rectangular space ABCD, represents a plate of mica standing in a vertical position. When a prism of Iceland spar is placed in a vertical or a horizontal line upon this plate, *polarised* light, viewed through them both, suffers no change. The horizontal and vertical lines EF, GH, therefore, drawn upon the plate of mica, may be called the *neutral* axes of the mica. When the Iceland spar is placed in the diagonals AD, BC of the rectangular plate, so as to bisect the right angles formed by the neutral axes, the *polarised* light is *depolarised*, and hence these dia-

gonals may be called the *depolarising* axes. These two axes are common to all substances that have the faculty of depolarisation.

If we now examine a *polarised* image by the prism of Iceland spar, placed upon the *vertical neutral* axis of the mica, the polarity of the light will of course continue, and only one image will be seen; but if we incline the plate of mica *forwards*, so as to make the polarised light fall upon it at an angle of about 45°, the image that was formerly invisible starts into existence, and therefore the light from which it was formed has been depolarised. If the same experiment is made upon the *horizontal neutral* axis, no such effect is produced: and hence it follows, that the *vertical neutral* axis is accompanied with an *oblique depolarising axis*. By making the same trials with the depolarising axes, it will be found, that each depolarising axis is accompanied with an *oblique neutral axis*; and therefore, each plate of mica possesses *two oblique neutral* axes, and only *one oblique depolarising* axis. The oblique depolarising axis of the mica is represented in Plate XI. Fig. 7. by the line On, and the two oblique neutral axes by the lines Om and Op; the angles GOn, AOm, BOp being about 45°, and the planes of these angles being perpendicular to the plate of mica.

In repeating these experiments with a great variety of transparent bodies, I have not been able to detect in any of them, except *topaz*, the oblique axis of depolarisation and neutrality. This mineral, which is known to have the laminated structure of mica, possesses only the *oblique depolarising axis*, which seems to indicate a less complicated structure than that of mica.

If we attend carefully to the two images of a luminous object, when they are *depolarised* by a plate of mica, they will exhibit, by a gentle inclination of the plate, the most capricious alternation of the prismatic colours. The *red* rays of the spectrum go to the formation of one image, while the *blue* rays go to the formation of the other. By a slight change in the obliquity of the plate, the *red* image becomes *blue*, and the *blue* image *red*, and these alternations are produced in such sudden fits, as to resemble more the tricks of a juggler, than the operations of a natural cause.

When one plate of mica is laid upon another, so that the *neutral* axis of the one may coincide with the *depolarising* axis of the other, all the *neutral* axes are converted into *depolarising* axes; and the play of prismatic colours appears in every position of the Iceland spar. Hence it follows,

that this singular decomposition of light is some how or other connected with the property of depolarisation, for the colour never appears in any of the neutral axes.

I have also observed this alternation of colour in the topaz, but only at the particular instant when the evanescent image started into view by the sudden restoration of the depolarising virtue; and it is also worthy of particular notice, that the colours never appear in very thin plates of mica.

4. *On some new Optical Phenomena observed in a Rhomboid of Iceland Spar.*

In the course of my experiments upon double refraction, I obtained from Sir George Mackenzie, Bart. some excellent crystals of Iceland spar, which exhibited a number of very curious phenomena, that were quite new to me, and which appear to be connected with the phenomena in the preceding Section, and with the colours of thin plates first observed by Newton. As I cannot pretend to explain these appearances, I shall satisfy myself at present, with giving an account of the experiments which I made.

The rhomboid of Iceland spar is represented in

Plate XI. Fig. 8. It is about 2-10ths of an inch thick, and the face AB forms an angle of about 135° 52′ with the surface CD. When we look at a candle through the parallel planes of the rhomboid in any part of the rectangular space *m n*, three images are distinctly visible, as in Fig. 9, where A is the common image of the candle, apparently single, and *d, e* the other images placed at equal distances from A.

If the side AB is brought towards the eye, the whole rhomboid moving about CD as an axis, the images *d, e* separate from A, and *e* becomes *yellow, red, purple,* and *blue,* in succession, by increasing the angle of incidence. The image *d* becomes *yellow* and *red,* and it vanishes before it assumes any other colour. When the side EF is brought near the eye by a motion round CD, the images *d, e* approach to A, but suffer no change of colour. When the side FB is brought near the eye by a motion of the rhomboid round MN, the images *d, e* approach to A, and become *yellow, red, purple,* and *blue,* in succession.

When the side EA is brought nearer the eye by a motion of the rhomboid round MN, the images *d, e* separate from A, but do not exhibit any alternation of colour: they are only more affected with the usual prismatic colours.

If the image of a taper polarised by reflection from water is examined by looking through the rectangular space *m n* of the rhomboid, the image *d* vanishes when the line bisecting the angle *m n* F is in the plane of reflection, and the image *e* vanishes when the line bisecting the angle *n* FE is in the plane of reflection.

When the images *d*, A, *e* are examined through another prism of Iceland spar, each of them is doubled; and, consequently, six images are visible. If the principal section of the spar is perpendicular to the line *d*, A, *e*, and if all the images are in a line, the image *e*, and the second image of *d*, vanish together; but if the principal section of the crystal is parallel to *d* A *e*, the image *d*, and the second image of *e*, vanish at the same instant.

If we now look at the candle through the inclined face AB, we see, as in Fig. 10. the images B, C, which are the double image A, separated by a greater refraction, and also the images *d*, *e* as before.

By examining the polarised light of a taper as before, the images *d* and C vanish when the line bisecting *m n* F is in the plane of reflection, and the images B and *e* vanish when the line bisecting *n* FE is in the plane of reflection.

When the images *d*, B, C, *e*, are viewed through a prism of Iceland spar, the images *e*, C, the second image of B, and the image *d*, all vanish in the same position as when *e* vanished in Fig. 9; and the second images of *e* and C, the image B, and the second image of *d*, vanish together in the same position as when *d* vanished in Fig. 9.

When the side AB is brought towards the eye by a motion round CD, the image *d* becomes yellow, red, purple, and blue, in succession: The image *e* is also affected with these colours, and the image C in a less degree, but the image B is never affected.

All the phenomena which have now been described, are visible in small crystals, about the 60th part of an inch in thickness, when detached from the farthest side of the rhomboid about M.

In the specimen of calcareous spar with which the preceding experiments were made, the prismatic face AB is cut in the same direction with the line *d e* which joins the images; but I obtained another specimen, in which the prismatic face was at right angles to that line. This specimen is represented in Fig. 11. of Plate XI. and exhibits some phenomena which are worthy of notice. The angle DCG is about $41\frac{1}{2}°$, and the inclination of the lines CD, FG is nearly $63\frac{1}{2}°$.

When we look at the candle through the parallel planes B b e E, ACGH, three images are seen as in Fig. 9. the line d A e being parallel to B b, and when the candle is viewed through the parallel faces BAHE, CDFG, the three images are seen as before; but d and e are at a greater distance from A, and the line d e is parallel to a line Ff, perpendicular to CG.

If the candle is examined through the inclined planes N b DC, NPGC, only two images are seen, as in every other specimen of calcareous spar; but when it is viewed through the planes AB b N, AHPN, so as to appear through the section NMOP, four images of the candle will be visible as in Fig. 12. where B, C are the common double images, and d, e the other images, of which e appears to be a second image of B, and d a second image of C. By turning the prism round so as to bring BD towards the eye, the images d, e recede from B, C, and by bringing AC towards the eye, these images approach to BC.

The images d, e, exhibit the same alternation of colours, and the same phenomena by polarisation which have already been described as seen through the prismatic face AB of the first specimen; and they are distinctly visible through an aperture of the 130th of an inch, when placed on any of the prismatic faces.

When the direction of a window bar is KL, the image of it, corresponding to *e*, has no colour at its edges; and when the direction of the window bar is RS, the image of it, corresponding to *d*, is not coloured. When the two common images of the bar are colourless, NP is the direction in which it lies.

5. *On the Modification of Light reflected from the Oxidated surface of polished Steel.*

As it appeared from the experiments of Malus, that light was not polarised by reflection from polished metallic substances, I wished to ascertain the effects of bodies approaching to the metallic state, and with this view I began a series of experiments upon the oxidated surfaces of polished steel. The colours produced upon polished steel at different temperatures, were ascribed by Sir Isaac Newton to the same cause as the colours of thin plates, and he supposed, that the different colours were occasioned by different thicknesses of a *thin glassy film*, arising from the *scoriæ or vitrified part of the metal being protruded and sent out to the surface.* This explanation, though marked by that wonderful sagacity which characterised

even the conjectures of this illustrious philosopher, had still no direct evidence to support it; and it was manifest, that the experiments which I proposed, would either confirm or overturn the hypothesis. If the light reflected from the oxidated surface experienced no modification, the existence of a transparent film might have been fairly questioned; while the communication of *polarity* to the reflected rays, which was impressed by every other transparent body, would give new strength to the conjecture of Newton.

Having procured several pieces of steel highly polished, I obtained, at different temperatures, all the shades of colour from a pale straw to the deepest indigo. When the light of a taper was reflected from the indigo coloured oxide,* at a great angle of incidence, (about 75° or 80°) and examined by a prism of calcareous spar, it did not seem to have received any new modification; but when the angle of incidence was diminished to about 55° and 60°, the least refracted image was of a brilliant red colour, while the other image did not appear to have suffered any change. By turning the prism about the axis of vision, the greatest

* The oxide most proper for making this experiment is formed at the temperature of 570°, according to Mr Stoddart's experiments.

refracted image was a bright red at the end of one fourth of a revolution, and at every quarter of a revolution the images were affected alternately with this brilliant colour.

When the reflected light was viewed through a plate of agate, having its laminæ perpendicular to the plane of reflection, the image of the taper was of a bright red, and the unformed nebulous light with which it was surrounded was a pale blue; but when the laminæ were parallel to the plane of reflection, the image assumed its usual colour.

By interposing a plate of mica between the blue oxide and the Iceland spar, or the plate of agate, the red image was restored to the same colour as the other image.

When polarised light produced by reflection from water, or by transmission through the agate, was reflected at a particular angle from the blue oxide, and in a plane perpendicular to the plane of reflection, the image appeared of a red colour, similar to that which was produced in the other experiments.

By using the other pieces of steel, in which the colours were produced at lower temperatures than the indigo coloured oxide, the image, which in the latter case became red, now varied from orange to yellow.

These curious results, while they appear to establish Sir Isaac Newton's idea of the transparency of the superficial film, seem, upon that supposition, to admit of an easy explanation. The light which is reflected from the transparent oxide becomes completely polarised at a particular angle of incidence, like the light reflected from every other diaphanous body; but the light which is transmitted through this film, and reflected to the eye from the polished steel having suffered almost no polarisation, the polarised portion will of course vanish in every quadrant, while the unpolarised portion will reach the eye of the observer. The truth of this explanation is strengthened by the result which was obtained by the interposition of mica; for when the polarised portion is *depolarised*, or restored by the mica, the image which was formerly red resumes its usual appearance.

6. *On the Modification of Light reflected from polished Metallic Surfaces.*

In the course of the preceding experiments, when the light reflected from the oxide was depolarised by the mica, I observed the same alternations of colour which have already been de-

scribed, and the colours still exhibited themselves, though with diminished effect, even when the polished steel had no oxidation upon its surface. These colours generally appeared between 45° and 70° of incidence, and I observed them when the reflection was made, from the following substances, which were the only metallic bodies that I tried:

Gold,	Metal for specula,
Silver,	Silvered back of a looking glass.
Brass,	Mercury,
Steel,	Pyrites.

From these observations, it follows, contrary to the assertion of Malus, that light does suffer some modification by reflection from metallic surfaces. Since none of the two images vanishes when the light is viewed through the Iceland spar, it is obvious, that the whole of the incident light cannot have been polarised; but a portion of it may have suffered this modification, and this polarised portion, when depolarised by the mica, may have been the cause of those alternations of the red and blue rays which, as we have already seen, uniformly accompany depolarised light.

7. *On the Light reflected from the Clouds; the Blue Light of the Sky; and the Light which forms the Rainbow.*

We have already seen, that the singular decomposition of light produced by the intervention of a plate of mica, is exhibited only when the transmitted rays have been previously polarised. This alternation of the prismatic colours, therefore, may be assumed as a decisive test, that the light by which they are formed has received either wholly or partly the character of polarisation; and by thus distinguishing reflected from direct light, it enables us to account for several interesting phenomena which have hitherto been only hypothetically explained.

When we examine the *light of the clouds* by a prism of Iceland spar, and interpose a plate of mica, the alternation of the prismatic colours is distinctly visible, although none of the two images formed by the spar vanishes in every quadrant. It follows, therefore, that the light of the clouds is partly polarised.

When the *blue light of the sky* is examined in a similar manner, the play of the prismatic co-

lours is still more brilliant than in the preceding experiment; and one of the images suffers a visible diminution of brightness at every quarter of a revolution. Hence we may conclude, that the blue light of the sky has experienced a partial polarisation; and that it is reflected from the atmosphere with which the earth is surrounded.

This result confirms Bouguer's hypothetical explanation of the blue colour of the firmament, and completely refutes the opinion of Fromondus, Otto Guericke, Wolfius, and Muschenbroek, who maintained, that the blue colour arose from a mixture of light and shade. The notion of Dr Eberhard, that the air has a proper colour of its own, is also overturned by the preceding experiment.

Upon examining with a prism of Iceland crystal the light of a very brilliant rainbow, I was surprised to find, that one of the images of the coloured arch alternately vanished and re-appeared in every quadrant of the circular motion of the prism. The light, therefore, which forms the bow has been almost wholly polarised; and when we recollect, that this light has been reflected from the interior surfaces of the drops of rain, nearly at the angle at which light acquires this property, the phenomenon admits of an easy explanation.

I have not yet found leisure to extend these experiments to the light of all the planetary bodies. The light of *Sirius* exhibits no marks of polarisation; and I have not been able to perceive any peculiarity in the rays of the full moon. As the light of the star is not borrowed from any other luminary, this result is precisely what might have been expected; and as the light of the full moon is reflected perpendicularly from her disc, the polarisation is in this case evanescent, and could not, therefore, be rendered visible.

I expect, however, to find, that the light of the moon, when she is about three, or twenty-seven days old, has received a partial polarisation, and thus to ascertain that she is directly and exclusively illuminated by the solar rays. Ricciolus maintained, about the middle of the 17th century, that the lunar surface is phosphorescent, and that her native light is disengaged by the excitation of the solar beams; and the same hypothesis has been recently published by Professor Leslie, to explain the lucid bow of light which bounds the obscure portion of the lunar disc. *

* The true explanation of this phenomenon, as deduced from direct observation, will be found in the article ASTRONOMY, in the *Edinburgh Encyclopædia*, vol. ii. p. 624.

The experiment which I have proposed, will decide the fate of these vague hypotheses, which derive no support either from direct or analogical reasoning.

The principle employed in the preceding experiments, furnishes us with a method of discovering whether there are any seas in the moon and planets. If the dark portion of the lunar disc is covered with water, as many astronomers have imagined, the light which they reflect must be *wholly* polarised, when the angle formed by lines drawn from the moon to the earth and the sun is 105° 39′; and this polarity will be indicated, either by the vanishing of one of the images at every quadrant of the motion of the Iceland spar, or by the alternation of the prismatic colours when a plate of mica is interposed.*

These experiments, which I am anxious to perform when the state of the weather is favourable, may be extended to the examination of the aurora borealis, and to any other luminous object with whose origin and nature we are unacquainted.

* A thin plate of transparent agate is peculiarly fitted for these observations when the light to be examined has a considerable intensity, as it polarises the transmitted ray without refracting it from its original direction, and, consequently, without producing the prismatic colours.

BOOK V.

ON

NEW TELESCOPES

AND

MICROSCOPES.

CHAP. I.

Account of an Improvement on Achromatic Telescopes, founded on a diminution of the secondary Spectrum which remains after equal and opposite Dispersions, and deduced from Experiments on the Action of Refracting Media upon the differently coloured Rays.

IT appears from the experiments contained in the Chapter on Dispersive Powers, that when the dispersion of a prism of crown glass is corrected by another of flint glass, the colour produced by the one prism is not wholly corrected, and that this uncorrected colour is indicated by a green and a wine coloured fringe surrounding the object from which the rays proceed. This important fact,

which was first observed by Clairaut, is completely hostile to the perfection of the achromatic telescope, and renders it impossible to correct more than two of the prismatic colours, by means of the opposite dispersions of two kinds of glass.

The celebrated Boscovich, whose fine and comprehensive genius has never been duly appreciated, confirmed the observation of Clairaut, by a series of well-conducted experiments. He shewed, that the uncorrected fringes arose from an inequality in the coloured spaces of the prismatic spectra formed by different substances; and he has pointed out a method of uniting three of the colours by three media, differing in refractive and dispersive power.

Dr Blair observed the uncorrected colour, which he calls a secondary spectrum, in several fluids: He found, that the muriatic and nitrous acids possessed the property of producing a spectrum in which the coloured spaces have a proportion different from that of all other substances, and he has shewn how to remove the uncorrected colour by a double combination of fluid lenses. This method, however, though the principle of it is perfectly correct, has never been used by subsequent opticians, and probably will never be found of any practical utility.

In order to explain the phenomena of uncor-

rected colour, let RRO, Plate XII. Fig. 1, be a ray transmitted through a hole in the window-shutter SS, and incident upon a prism P, which refracts the light in the direction PM, and forms on the wall OA the prismatic spectrum AB; PM being the mean ray, PB the extreme red, and PA the extreme violet ray. The spectrum AB, which it forms, will be composed of four colours, red, green, blue, and violet, and if the prism is made of crown glass, the mean ray PMN, which divides the spectrum, will be the boundary of the blue and green spaces. If a prism of flint glass, with a less refracting angle, is placed so as to form a spectrum CD of the same size with AB, the boundary *mn* of the green and blue spaces will no longer be the mean ray of the spectrum, but will be considerably nearer the red extremity D. Hence the least refrangible rays will be more *contracted*, and the most refrangible rays more *expanded* than in the spectrum AB. If a third spectrum EF of the same length with the other two is formed by a prism of rock crystal, the mean ray $\mu\nu$ will be nearer the violet extremity of the spectrum, and the least refrangible rays will be more expanded, and the most refrangible ones more contracted than in the spectrum formed by the crown glass.

These conclusions respecting the different magnitudes of the corresponding colours, are not obtained by examining with the eye the length that each colour occupies in the spectrum. The indistinctness with which all the coloured spaces are bounded renders this quite impracticable, and compels us to adopt a more circuitous, though, at the same time, a more accurate method of observation.

If a spectrum formed by flint glass had its coloured spaces exactly of the same dimensions with those of an equal spectrum formed by crown glass, any object, such as the sun, or a window bar lying parallel to the common section of the refracting planes, should appear perfectly colourless when seen through the combined prisms. But if the coloured spaces in the two spectra are not proportional, as is represented at AB, and CD, Fig. 1. Plate XII. then the sun, or the window bar, cannot be altogether free from colour; for though the extreme red and violet rays of both the spectra are united, yet the intermediate colours are not rendered coincident. In the spectrum AB, formed by the crown glass, the first green ray MN, which is here the mean ray, is obviously more refracted than the first green ray *m n*, in the spectrum CD formed by the flint glass,

and therefore the flint glass will not be able to refract the green ray, so as to unite it with the red and violet. Hence the green ray will, as it were, be left behind, while the red and violet rays are rendered coincident. Thus, in Fig. 1. if a prism *p* of flint glass is placed behind a crown glass prism P, so as exactly to correct its dispersion, the spectrum AB will be reduced to a secondary spectrum *a b*, the upper half of which is green, which is left behind, and the lower half is of a wine colour formed by the union of the red and violet rays. If the bar of a window had been examined through the combined prisms P, *p*, the upper side of it would have been tinged with green, and the lower side of it with a wine-coloured fringe.

By comparing, in a similar manner, the spectrum EF formed by rock crystal, with the spectrum AB formed by crown glass, it will be found, that the rock crystal having a greater action than the crown glass upon the green ray, will carry it beyond the place of the united red and violet, and will form a secondary spectrum *e f*, the lower half of which is green, and the upper half of a wine colour, arising from the union of the red and violet light. If the bar of a window were viewed through the combined prisms of crown glass and rock crystal, it would be tinged with green on its

lower side, and with a wine-coloured fringe on its upper side.

When a horizontal window bar, therefore, is seen through any two prisms which correct each other's dispersion, without uniting all the colours, the green fringe will always be on the same side of the bar with the vertex of the prism which has the least action upon the green light, or which *contracts* the red and green rays, and *expands* the blue and violet ones; that is, if the vertex of the flint glass prism is pointing downwards, the uncorrected green fringe will be on the lower side of the bar. By observing, therefore, the position of the green fringe, we can immediately ascertain which of the two prisms has the greatest action upon the green light.

These theoretical deductions from the assumed inequality of the coloured spaces, are completely established by observation. Prisms of crown and flint glass, having large refracting angles, and correcting each others dispersion, uniformly give a secondary spectrum like *a b*, Fig. 1; and for the same reason, if we look at the moon, or any luminous body, through the most perfect achromatic telescope that can at present be constructed, and draw out the eye-piece beyond the point of distinct vision, the moon will be encircled

with a brilliant margin of green light, and if the eye tube is pushed as far inwards between the point of distinct vision and the object-glass, the moon will be surrounded by a less brilliant fringe of wine-coloured light.

The existence and the origin of the uncorrected colour being thus clearly established, it becomes highly interesting to enquire into the cause of this irrationality of the coloured spaces. The improvement of the achromatic telescope, indeed, is involved in the inquiry, and it depends upon the conclusions to which we arrive, whether we shall obtain an approximate correction for all the remaining errors of the instrument, or abandon every hope of its future improvement.

Boscovich, Dr Blair, and Dr Robison, the only philosophers who have written upon this subject, maintain, that the inequality of the coloured spaces in different spectra, arises from a particular quality inherent in the bodies by which they are formed; and they suppose, that these bodies possess this quality in different degrees, in the same manner as they differ from each other in refractive and dispersive power. "If the case of unproportional dispersion," says Dr Blair, "should be found to hold true in fact, we shall arrive at this new truth in optics, that though in the re

fraction of a pencil of solar light, made in the confine of any medium, and a vacuum, the deep red rays are always the least refrangible, and the violet rays are always the most refrangible; yet it depends entirely on the specific qualities of the medium, which shall be the mean refrangible ray; the very same ray which in the refraction through one medium is the mean refrangible ray, being found in others among the less refrangible rays."

"There is no doubt among naturalists," says Dr Robison, "about the mechanical connection of the phenomena of nature; and all are agreed, that the chemical actions of the particles of matter are perfectly like in kind to the action of gravitating bodies; that all these phenomena are the effects of forces like those which we call attractions and repulsions, and which we observe in magnets and electrified bodies; that light is refracted by forces of the same kind, but differing chiefly in the small extent of their sphere of activity. One who views things in this way will expect, that as the actions of the same acid for the different alkalies are different in degree, and as the different acids have also different actions on the same alkali, in like manner different substances differ in their general refractive powers,

and also in the proportion of their action on the different colours. Nothing is more unlikely, therefore, than the proportional dispersion of the different colours by different substances; and it is surprising that this inquiry has been so long delayed."

When this subject first excited my attention, I imagined that the inequality of the coloured spaces was produced solely by a difference in the refracting angles of the prisms by which the equal spectra were formed;—an opinion which, to a certain extent, was founded on unquestionable principles. But though I have been driven from this hypothesis by the result of every experiment, yet it will be seen, in the sequel of this Chapter, that while an inequality in the coloured spaces, and consequently a *secondary spectrum,* is produced by a difference of action upon the different colours, there is also another inequality in the colorific intervals of an opposite nature, and consequently another *secondary* spectrum arising from the different circumstances under which the primary spectra are formed.

As the existence of this new secondary spectrum, or *tertiary* spectrum, as it may be called, may be deduced from optical principles, we shall first consider, theoretically, the changes produced

upon the coloured spaces by the different circumstances under which the spectrum is formed, and then compare these deductions with the results of experiment.

Now, there are four different ways in which a spectrum of a given length may be produced by different bodies.

1. The given spectrum may be formed by a prism of a substance, which has a very high dispersive, and a comparatively low refractive power. Such a prism, therefore, will produce the given spectrum by a small refracting angle. A spectrum is formed, in this manner, by oil of cassia.

2. A spectrum of the same length may be formed by a prism of a substance that has a very low refractive power. It must therefore be produced by a very great refracting angle. This will be the case with fluor spar.

3. A spectrum of the same length may be formed by a body which has a low dispersive power, and a very high refractive power. A small angle, therefore, will only be necessary in this case. This will happen when the spectrum is formed by diamond.

4. A spectrum, of a given length, may be formed, by diminishing the angle of incidence at the first

surface of the prism, so that the greatest refraction may be produced at the second surface.

There are various intermediate circumstances, approaching more or less to one of these four cases, under which a similar spectrum may be produced, as in the case of topaz, which has a great refractive power, but where the dispersive power is so very low as still to require a large refracting angle for the formation of the spectrum. These cases, however, do not require to be particularly noticed.

In order to understand the variations which the spectrum undergoes when produced under different circumstances, let AO, Plate XII. Fig. 2, be a ray of light incident almost perpendicularly upon the refracting surface EF, and let it be deflected from its original path AOP, and separated into its component parts, so that OR may be the extreme red or least refrangible ray, and OV the extreme violet or most refrangible ray. Bisect the angle ROV by the line OM, and from the points R, M, V draw the lines R*a*, M*b*, V*c* perpendicular to OS, and AB perpendicular to OT. Then if EF is the surface of a glass medium, of which the index of refraction is 1.548, we shall have, when AB is equal to 12, $Ra=7.80$, $Mb=7.75$, and $Vc=7.70$. As the sines do not increase uniformly, but suffer a gra-

dual diminution, while the corresponding arches are regularly augmented, the differences of the sines of equidifferent arcs are not equal, and therefore the value of Mb must be a little greater than 7.75; but as the arches RV and MS decrease, this inequality in the differences of the sines gradually diminishes. We may therefore safely assume, that in small angles of incidence, Ra—Mb=Mb—Vc, and consequently that a ray OM, whose sine of refraction is 7.75, bisects the angle ROV formed by the extreme rays, and is therefore the mean refrangible ray of the spectrum RV.

If another ray of light CO, moving in the direction COp, falls upon the same surface at O, with a very great angle of incidence, it will form the spectrum rv considerably larger than RV. Then if we draw from the points C, r, and v, the lines CD, rd, and vf perpendicular to TOS, and suppose CD to be 48, or quadruple of AB, we shall have, from the constant ratio of the sines of incidence and refraction, $rd=7.80\times4=31.12$, and $vf=7.70\times4=30.80$. The ray MO, which in the former refraction was the mean refrangible ray, will have its new sine of refraction $me=7.75\times4=31.00$, and therefore, from the retardation in the increase of the sines, the point m will not be equidistant from r and v, that is, the ray Om, which was for-

merly the mean refrangible ray of the spectrum, will now be nearer the extreme violet Ov, while one of the least refrangible rays will bisect the spectrum, or have become the mean refrangible ray. Hence it follows, that in all refractions from *a rare into a dense medium, the* RED *and* GREEN, *or the least refrangible rays, are expanded by increasing the angle of incidence, while the* BLUE *and* VIOLET, *or the most refrangible rays, are contracted.*

The contraction and dilatation which are thus experienced by the most and the least refrangible rays, may be easily found by determining the deviation of the mean ray Om, from the line OL, which bisects the spectrum $r\,v$. For this purpose, let

$a =$ Angle of incidence COT.
$r =$ Index of refraction for the extreme red ray.
$v =$ Index of refraction for the extreme violet ray.
$x =$ Angle of refraction for the extreme red ray.
$y =$ Angle of refraction for the extreme violet ray.
$z =$ Angle of deviation of the mean ray $= m$ OL.

Then, in refractions from a rare to a dense medium, we have

$$r : 1 = \text{Sin. } a : \text{Sin. } x, \text{ and}$$

$$\text{Sin. } x = \frac{\text{Sin. } a}{r}$$

$$v : 1 = \text{Sin. } a : \text{Sin. } y, \text{ and}$$

$$\text{Sin } y = \frac{\text{Sin. } a}{v}$$

But from the constant ratio of the sines, the ray OM, which was the mean refrangible ray at a very small angle of refraction, will have its sine an arithmetical mean between Sin. x and Sin. y. Hence,

$$\frac{\text{Sin. } a}{2\,v} + \frac{\text{Sin. } a}{2\,r}, \text{ which we may call sin. } \varphi,$$

will be the sine of the ray that was formerly the mean ray. Since, therefore,

$$\varphi = \text{the angle } m\,O\,v, \text{ and}$$
$$\frac{x+y}{2} = v\,O\,L = r\,O\,L, \text{ we have}$$
$$z = \frac{x+y}{2} - \varphi = m\,OL$$

If we suppose

$$\begin{aligned}
a &= 90°, \text{ so as to have the greatest refraction.}\\
r &= 1.55842\\
v &= 1.53846, \text{ we shall have}\\
x &= 39°\ 55'\ 0''.8\\
y &= 40\quad 32\quad 29.8\\
y - x &= \ \ 0\quad 37\quad 29.0\\
\frac{x+y}{2} &= 40\quad 13\quad 45.3\\
\varphi &= 40\quad 18\quad 42.7, \text{ and}\\
z &= 2''.6
\end{aligned}$$

The red and green rays, therefore, will now subtend an angle of 18′ 47″.1, while the blue and violet subtend only an angle of 18′ 41″.9.

If EF, Fig. 9. is the boundary of a rare medium such as air, and if the refraction is made from glass, it will be found by constructing the figure for this

new case, that the mean refrangible ray approaches to the red extremity of the spectrum, as the angle of refraction is increased; and hence it follows, that in all refractions, from a *dense into a rare medium, the red and green, or the least refrangible rays, are contracted* by increasing the refraction, while the *blue and violet, or the most refrangible rays, are expanded.*

By employing the same symbols that were used in p. 365, we shall have, in the case of a refraction from a dense into a rare medium,

$$1 : r = \text{Sin. } a : \text{Sin. } x, \text{ and}$$
$$\text{Sin. } x = \text{Sin. } a \times r$$
$$1 : v = \text{Sin. } a : \text{Sin. } y, \text{ and}$$
$$\text{Sin. } y = \text{Sin. } a \times v.$$

Hence,

$$\text{Sin. } \frac{a \times r}{2} + \text{Sin. } \frac{a \times v}{2} = \text{Sin. } \varphi, \text{ and}$$

$$z = \frac{y + x}{2} - \varphi.$$

Assuming

$a = 40° \; 32' \; 29''.8$, so as to have the greatest possible refraction for the blue rays.

$v = 1.53846$

$r = 1.55842$, we shall have

$y = 90° \; 0' \; 0''$

$x = 80° \; 47' \; 3''.6$

$\frac{x + y}{2} = \varphi = 85° \; 23' \; 31''.8$

$\varphi = 83° \; 29' \; 13''.5$, and

$z = 1° \; 54' \; 18''.3$, or the maximum deviation of the mean refrangible ray.

The red and green half of the spectrum will consequently subtend an angle of 2° 42′ 9″.9, while the blue and violet half subtends an angle of 6° 30′ 46″.5.

Since the angles of refraction of the extreme red and violet rays are always greater than the angle of incidence when the refraction is made from a dense into a rare medium, and less than that angle when the refraction is from a rare into a dense medium, it follows, that when a pencil of light is refracted at a given angle of incidence from a rare into a dense medium, the *expansion of the red rays will be less than the expansion of the blue rays, or the corresponding contraction of the red rays, when the refraction is made at the same angle of incidence from the denser into the rarer medium.*

The contraction of the least refrangible rays, and the corresponding dilatation of the most refrangible rays, when the light passes into a rarer medium, will be more easily understood from Fig. 3. Plate XII. where CDE is a prism of glass, and AB a ray of light incident at B, with such an angle that the mean ray Bm, which bisects the angle v B r, formed by the extreme red and violet rays, in consequence of the refraction at B, shall fall perpendicularly upon the second surface ED. In

this case, the extreme violet ray Bv will fall upon the second surface, at the same angle of incidence with the extreme red ray Br: The mean ray Bm, will pass on in the direction mM, without suffering any refraction; and as the violet ray Bv is more refrangible than the red ray Br, it will be more deflected from its original path B a, than the red ray will be from its path B b; and consequently the angle V v a being greater than R r b, and a BM = b BM, the ray BM, which was the mean ray after the first refraction, will be nearer to the red extremity R of the spectrum, and the least refrangible rays will have experienced a contraction, and the most refrangible rays a corresponding degree of dilatation.

When the light, therefore, is transmitted through a prism, the red rays will be expanded at the first refraction, and contracted at the second; but as the angle of refraction is almost always greater at the second surface, and as the spectrum is produced chiefly by the second refraction, the contraction of the red rays will always greatly exceed their dilatation. Hence it follows, that, *by refraction through a prism, the red rays are contracted and the violet rays expanded, and their contraction and dilatation increases as the refracting angle of the prism is augmented.*

From similar reasoning we may conclude, that the contraction of the red and the expansion of the violet rays by any prism, will be increased by making the incident rays fall more perpendicularly upon the first surface, so that the greatest refraction may be produced at the second surface.

Let us now consider the nature of the spectrum produced by a high refractive power, as in the case of diamond and some of the precious stones; and let us compare it with an equal spectrum produced by a large refracting angle, the dispersion in both cases being supposed the same. Upon the surface EF, Plate XII. Fig. 4. of a dense medium, with a low refractive power, let a ray of light CO*p* be incident at O, with a very great angle, so as to form, in consequence of the magnitude of this angle, the spectrum *r m v*. The red and green rays in this spectrum will be obviously expanded, and the blue and violet ones condensed. If EF is now supposed a medium of a very high refractive power, such as diamond, but having the same dispersive power as before, then, in order to produce a spectrum equal to *r m v*, the ray must be incident at a very small angle AOT. Since the dispersive powers are assumed equal in both cases, the angles of deviation *r o p*, ROP,

and $v o p$, VOP, will be equal; and therefore RV will be the spectrum formed by the high refractive power. But in the spectrum RV, owing to the unequal increase of the sines, the mean refrangible ray, whatever it is, will be nearer R than in the spectrum $r v$; that is, the ray OM, which bisects ROV, will have the position O m nearer to v than to r. It follows, therefore, when the refraction is from a rare into a dense medium, that *in a spectrum formed by a high refractive power, the* RED *and* GREEN *rays will be less expanded, and the* BLUE *and* VIOLET *less contracted than in an equal spectrum produced by a great refracting angle of a substance with a low refractive power.*

If we construct the figure for the case where the refraction is made from a dense into a rare medium, it will be found, that *in a spectrum formed by a high refractive power, the* RED *and* GREEN *rays will be less contracted, and the* BLUE *and* VIOLET *less expanded, than in an equal spectrum, formed by a great refracting angle of a substance with a low refractive power.*

Although the preceding deductions are sufficiently clear and simple, yet, from the importance and difficulty of the subject, I was anxious to confirm them by the evidence of direct experiment.

The extreme delicacy of such experiments, however, arising from the almost evanescent magnitude of the uncorrected fringes, renders it difficult to obtain results that inspire confidence; and had I not fortunately detected the singular dispersing power possessed by oil of cassia, I should have abandoned the enquiry as beyond our means of investigation. To attempt to determine by the unassisted eye, the relative proportions of the coloured spaces in different spectra, is to use an instrument where the error of observation exceeds the actual result. To ascertain the relative magnitude of the uncorrected fringes produced by combinations of prisms of crown and flint glass, or other substances that do not differ very widely in dispersive power, is a measurement beyond the grasp of the most accurate observer; and to magnify the effect by forming the substances into achromatic object glasses, was to adopt a plan attended with great expence and labour, and applicable only to transparent fluids.

The high dispersive power of oil of cassia, which is nearly *seven* times greater than that of cryolite and fluor spar, and is accompanied with a comparatively low refractive power, relieves us from all these embarrassments, and gives us the command of a scale of very unusual magnitude. I

therefore considered the proportion of the coloured spaces in a spectrum produced by this oil as a standard to which their proportions in other spectra might be referred; and, with a few exceptions, which will afterwards be stated, I have uniformly found, from the magnitude and position of the uncorrected fringes, that in the spectrum produced by bodies with a high dispersive power, the least refrangible or red rays are most contracted, and the violet rays most expanded; that with prisms of the same substance, the red rays are most contracted by a large refracting angle; and that this contraction is still farther increased, by diminishing the angle of incidence at the first surface.

First Series.

Exp. 1. When the dispersion of a prism of water, with a refracting angle of about 68°, is corrected by a prism of oil of cassia, with a refracting angle of about 8° 16′, the bar of the window, when viewed through the combined prisms, is tinged on one side with a very broad fringe of wine-coloured light, and on the other with a similar fringe of a brilliant green, the green fringe being on the same side of the bar with the vertex of the oil of cassia prism.

Exp. 2. When the dispersion of a prism of

crown glass having a refracting angle of about 41° 11′ is corrected by a prism of oil of cassia, with a refracting angle of 8° 16′, the uncorrected fringes are a little narrower than in Exp. 1. and the green fringe is on the same side of the bar as the vertex of the oil of cassia prism.

Exp. 3. When the dispersion of a prism of flint glass, with a refracting angle of about 23° 26′, is corrected by the prism of oil of cassia of 8° 16′, the uncorrected fringes are narrower than in Exp. 1. and 2. and the green fringe is on the same side of the bar as formerly.

Exp. 4. When the dispersion of a prism of rock crystal is corrected by a prism of oil of cassia, the uncorrected colour is greater than in Exp. 1, 2, and 3, and the position of the green fringe is the same as before.

Exp. 5. When a prism of rock crystal is corrected by a prism of flint glass, the uncorrected green is towards the vertex of the flint glass.

Exp. 6. When the dispersion of a prism of blue topaz is corrected by a prism of oil of cassia, the uncorrected colour is nearly the same as in the preceding experiment, the green fringe having still the same position.

Exp. 7. When the dispersion of a prism of fluor spar is corrected with a prism of oil of cassia,

the uncorrected colour is very great, the position of the green fringe being the same as formerly.

Exp. 8. When the dispersion of a prism of diamond is corrected by oil of cassia, the uncorrected colour is nearly as great as in Exp. 3. with flint glass, the position of the green fringe being the same.

Exp. 9. When a prism of opal coloured-glass is corrected by oil of cassia, the uncorrected colour is less than in any of the preceding experiments, the green fringe being on the same side of the bar as before.

Exp. 10. When a prism of balsam of Tolu is corrected with oil of cassia, the uncorrected fringes are much less than in any of the preceding experiments, the green fringe having still the same position.

If all the preceding experiments are repeated, by substituting balsam of Tolu instead of oil of cassia, the results will be nearly the same, with this difference only, that the uncorrected fringes will be narrower and less distinct.

Exp. 11. When a prism of crown glass has its refraction corrected by a prism of flint glass, the uncorrected green is towards the vertex of the flint glass prism.

The red and green spaces are, therefore, more

contracted in the spectrum formed by the flint glass, than in the spectrum formed by the crown glass.

Exp 12 When a prism of rock crystal is corrected with a prism of crown glass, the uncorrected green fringe is towards the vertex of the crown glass prism.

As the uncorrected green is always towards the vertex of the prism that gives the greatest contraction to the red and green rays, the spectrum formed by rock crystal has its red and green spaces more expanded than those of the crown glass spectrum.

Exp. 13. When the spectrum of a prism of muriatic acid is corrected by a prism of crown glass, the uncorrected green is towards the vertex of the crown glass.

Hence, the red and green spaces are more expanded in the spectrum formed by the muriatic acid, than in the spectrum formed by the crown glass.

Exp. 14. When muriatic acid has its spectrum corrected by a prism of rock crystal, the uncorrected green is towards the vertex of the prism of muriatic acid.

Hence it follows, that the red and green spaces

formed by the rock crystal, are more expanded than in the spectrum formed by the muriatic acid.

Exp. 15. When a prism of oil of lavender is corrected by a prism of flint glass, the uncorrected green is towards the vertex of the oil of lavender.

The red and green spaces are therefore more contracted in the spectrum formed by the oil of lavender, than in the spectrum formed by the flint glass.

Exp. 16. When a prism of oil of lavender is opposed by a prism of crown glass, the uncorrected green is towards the vertex of the oil of lavender, and the fringes are broader than in Exp. 15.

Exp. 17. When oil of lavender opposes balsam of Tolu, the uncorrected green is towards the vertex of the balsam of Tolu.

Exp. 18. When oil of lavender is corrected by oil of cassia, the uncorrected green is towards the vertex of the oil of cassia.

Exp. 19. When oil of sassafras opposes oil of cassia, the uncorrected green is towards the vertex of the oil of cassia.

Exp. 20. When oil of sassafras opposes balsam of Tolu, the uncorrected green is towards the vertex of the balsam of Tolu.

Exp. 21. When oil of sassafras opposes flint

glass, the uncorrected colour is towards the vertex of the oil of sassafras.

Exp. 22. When oil of sassafras opposes crown glass, the uncorrected green is towards the vertex of the oil of sassafras.

Exp. 23. When flint glass, with a refracting angle of 41°, is corrected by crown glass, with a refracting angle of about 69°, the uncorrected colours are remarkably distinct, and the green fringe is towards the vertex of the flint glass.

Exp. 24. When crown glass corrects a red coloured glass, the uncorrected green is towards the vertex of the red glass.

Exp. 25. When flint glass corrects red coloured glass, the uncorrected green is towards the vertex of the paste, though it is not easily seen.

Exp. 26. If balsam of Tolu is corrected by gum arabic, the uncorrected green is towards the vertex of the balsam of Tolu.

Exp. 27. When gum arabic opposes flint glass, the uncorrected green is towards the flint glass.

Exp. 28. When gum arabic is corrected by topaz, or by rock crystal, the uncorrected green is towards the gum arabic, the fringes being broader with rock crystal.

Exp. 29. When gum arabic is corrected by crown glass, the uncorrected colour is extremely

small, and the green appears to be towards the crown glass.

Exp. 30. When oil of cummin is corrected by flint glass, the uncorrected green is towards the oil of cummin.

Exp. 31. When oil of cummin opposes balsam of Tolu, the uncorrected green is towards the balsam of Tolu.

Exp. 32. When oil of cummin is corrected by oil of lavender, the uncorrected green is towards the oil of cummin.

Exp. 33. When calcareous spar (1st refraction) with a refracting angle of 63° 30′ is corrected by flint glass, with an angle of about 65°, the uncorrected green is towards the vertex of the flint glass.

Exp. 34. When calcareous spar (2d refraction), with an angle of about 65°, opposes crown glass with an angle of 41° 11′, or 69°, inclined so as to increase the dispersion, the uncorrected green is distinctly towards the vertex of the crown glass prism.

Exp. 35. When calcareous spar (2d refraction), with an angle of about 42°, opposes crown glass with an angle of 69°, the spar being inclosed so as to increase the dispersion, the uncorrected green is towards the calcareous spar.

Exp. 36. When calcareous spar (1st refraction), with an angle of about 42°, is inclined so as to equal the dispersion of crown glass with an angle of 69°, the uncorrected green is towards the calcareous spar.

Exp. 37. When calcareous spar (1st and 2d refraction), are corrected by rock crystal and by topaz, the uncorrected green is in both cases towards the calcareous spar.

Exp. 38. If leucite is corrected by flint and crown glass, the uncorrected green is towards the glass, and when corrected by topaz, the uncorrected green is towards the leucite.

Exp. 39. When beryl is opposed to flint and crown glass, the uncorrected green is towards the glass.

Exp. 40. When tourmaline opposes flint and crown glass, the uncorrected green is towards the glass.

Exp. 41. If borax is corrected by flint and crown glass, the uncorrected green is towards the glass.

Exp. 42. When selenite is opposed to flint and crown glass, the uncorrected green is towards the glass; and when opposed to topaz, the uncorrected green is towards the selenite.

Exp. 43. If citric acid is corrected by flint glass, the uncorrected green is towards the flint glass.

Exp. 44. When gum juniper is opposed to crown glass, the uncorrected green is towards the gum.

Exp. 45. When Canada balsam is corrected by flint glass, the uncorrected green is towards the balsam.

Exp. 46. When carbonate of lead is opposed to oil of cassia, the uncorrected green is towards the oil of cassia.

Exp. 47. When carbonate of lead is corrected by balsam of Tolu, the uncorrected fringes are very small, and the green one is towards the balsam.

Exp. 48. When sulphur is corrected by crown glass, the uncorrected colours are nearly as great as in Exp. 2. the green being towards the oil.

Exp. 49. When sulphur is corrected with oil of cassia, no uncorrected colour is visible. The angles of the prisms were in this experiment very small.

Exp. 50. When oil of cloves is corrected by flint glass, Canada balsam, and oil of lavender, the uncorrected colour is, in all these cases, towards the oil of cloves.

Exp. 51. When oil of cloves is corrected with oil of sassafras, and oil of cummin, the uncorrected green is towards these oils.

Exp. 52. When oil of turpentine is corrected by flint glass, the uncorrected green is towards the oil; but when it is corrected by oil of sassafras, oil of cloves, and Canada balsam, the uncorrected green is towards these three fluids.

Exp. 53. When sulphuric acid is corrected by crown glass, the uncorrected fringes are very great, the green being towards the crown glass.

Exp. 54. If the sulphuric acid is opposed by rock crystal, the uncorrected green is still very great, the green being towards the rock crystal.

Exp. 55. When water corrects the sulphuric acid, the uncorrected green is towards the water.

Exp. 56. When fluor spar opposes the sulphuric acid, the uncorrected green is towards the fluor spar.

Exp. 57. When muriatic acid corrects the sulphuric acid, the uncorrected green is towards the muriatic acid.

Exp. 58. When rock crystal is corrected by water, the uncorrected green, which is very small, is towards the rock crystal.

Exp. 59. When alcohol is corrected by water, and by muriatic acid, the uncorrected green is in both cases towards the alcohol; and when it is corrected with crown glass, the uncorrected co-

lours are scarcely perceptible, the green being rather towards the crown glass.

Exp. 60. When ether is corrected by alcohol, no uncorrected colour is visible.

Second Series.

Exp. 61. If a flint glass prism, with an angle of 41° 11′, is corrected with another prism of flint glass, with an angle of 60° 2′, the first being inclined to increase the refraction, uncorrected colours are distinctly visible, the uncorrected green being towards the vertex of the smaller prism.

Exp. 62. When a flint glass prism, with an angle of 38° 54′, is opposed to another prism of flint glass, of 66° 2′, the first being inclined to increase the dispersion, the uncorrected green is towards the vertex of the smaller prism. Both these prisms were made of the same piece of glass.

The two preceding experiments being repeated with various prisms of flint glass, I always found that the prism with the smallest angle, which was inclined in order to increase the dispersion, had the uncorrected green towards its vertex: and what was still more singular, the *colourless pencil was still considerably refracted from its original direction, by the prism with the largest refracting angle.*

Exp. 63. When a prism of rock crystal, with a refracting angle of 25° 28′, was corrected by a prism of rock crystal, with an angle of about 70°, the first being inclined in order to increase the dispersion, the uncorrected green is towards the vertex of the prism with the least angle, the colourless pencil being considerably refracted by the largest prism.

When the two prisms of rock crystal have their angles 25° 28′, and 41° 20′, there is still a very considerable balance of refraction in favour of the larger prism, after the dispersion is completely corrected.

Exp. 64. If a prism of plate glass, with a small angle, is corrected by rock crystal with a large angle, the first being considerably inclined, the uncorrected green is towards the vertex of the plate glass prism.

Exp. 65. When balsam of Tolu 8°, is inclined so as to be corrected by flint glass 66° 2′, the uncorrected colour is enormous.

Third Series.

Exp. 66. When crown glass, with an angle of 41° 11′, is inclined so as to be corrected by flint glass 66°, the uncorrected green is towards the vertex of the crown glass.

Exp. 67. If a prism of crown glass, with an

angle of 41° 11′, is inclined, so as to be corrected by flint glass 50° 28′, the uncorrected green is towards the vertex of the flint glass.

Exp. 68. If the same prism of crown glass is inclined, so as to be corrected by flint glass with an angle of about 62°, no uncorrected colour is visible.

Exp. 69. When a prism of rock crystal, with an angle of 25° 28′, is inclined, so as to be corrected by a prism of flint glass with an angle of 66°, the uncorrected green is towards the vertex of the rock crystal.

It appears, from the first series of these experiments, from No. 1. to No. 60. inclusive, that the coloured spaces have a different proportion in equal spectra, formed by almost all transparent bodies. This variation in the magnitude of the coloured spaces, is obviously produced by a difference in the action of the refracting substances upon the differently coloured rays; for the effect of an augmentation of the refracting angle of the prism, in contracting the red and green, and expanding the blue spaces, is of an opposite nature from that which is actually produced.

From a comparison of the different experiments, I have drawn up the following Table,

which will shew with tolerable correctness, the effect of different bodies upon the differently coloured rays. The substances are arranged inversely, according to their action on green light; that is, those at the top of the Table form spectra, in which the red and green rays are most contracted, and the blue and violet ones most expanded. In these spectra, therefore, the green rays are nearest to the red extremity, and are, consequently, less refracted in proportion to the refraction of the other colours.

TABLE, *shewing the Order of the Substances that form Spectra, in which the Red and Green Spaces are most contracted.*

Oil of cassia.	Oil of almonds.
Sulphur.	Crown glass.
Balsam of Tolu.	Gum-arabic.
Carbonate of lead.	Alcohol.
Oil of anise seeds.	Ether.
Oil of sassafras.	Leucite.
Opal-coloured glass.	Blue topaz.
Oil of cummin.	Fluor spar.
Oil of cloves.	Nitrous acid.
Oil of lavender.	Muriatic acid.
Canada balsam.	Rock crystal.
Oil of turpentine.	Water.
Flint glass.	Sulphuric acid.
Calcareous spar.	

From the preceding Table, the following conclusions may be drawn:

1. The action of transparent substances on the green rays, diminishes as their dispersive power increases.

2. The exceptions to this law are, oil of cloves, oil of lavender, Canada balsam, and oil of turpentine, which have a less action upon the green rays than flint glass, though they are inferior to that substance in dispersive power. The muriatic and the nitrous acids form another exception, as their action upon the green rays exceeds that of crown glass, though they surpass it in dispersive power. Fluor spar, rock crystal, water, and sulphuric acid, must also be considered as exceptions.

3. The sulphuric acid exceeds all transparent substances that have hitherto been examined in its action upon the green rays, while the oil of cassia exerts the least action upon them of any known substance. These two substances, therefore, might be employed to great advantage in the construction of fluid object glasses, in order to correct the secondary spectrum.

In performing these experiments, it is necessary that the prisms should have large refracting angles, in order to increase the uncorrected fringes; and,

for the purpose of magnifying them as much as possible, the window bar, as seen through the combined prisms, should be examined with a small telescope, having a magnifying power of ten or twelve times.

When the substances which are employed to correct each other's dispersion, differ considerably in dispersive power, a large angle of the one will sometimes be opposed to a small angle of the other, as in Exp. 56, where fluor spar is combined with sulphuric acid. In this case, the large angle of the fluor spar produces a small tertiary spectrum, which may either increase or diminish the secondary spectrum, and thus afford an erroneous result. This source of error, however, is too trifling to be noticed in general cases.

In repeating or extending the preceding experiments, a great variety of prisms with various refracting angles will obviously be necessary. This necessity, however, may in some measure be obviated, by finding the angle of one prism which will correct the spectrum formed by a given angle of another. Suppose, for example, that we wanted to observe the uncorrected fringes formed by prisms of crown and flint glass, and that we had a prism of crown glass with a given refracting angle: In order to form these fringes, the prism

of flint glass must have precisely that angle which will correct the dispersion of the crown glass. For this purpose we may employ an instrument similar to that which has been described in Book IV. Chap. III. and thus obtain the required angle by direct experiment. This instrument will also show us, whether or not there is a balance of refraction in favour of any of the two substances, and, consequently, if such a combination is fit for an achromatic object glass.

The same result may be obtained by calculation, in the following manner. If A is the given angle of one of the prisms, and α the required angle of the other substance necessary to correct the dispersion of the first, R the refraction of the first prism, and r that of the second; then we shall have

$$\text{Sin. } x' = \frac{R}{r} \times \text{Sin. A, and}$$

$$\text{Tang. } \overline{\alpha - x'} = \frac{dR}{dr \times \frac{R}{r}} - 1 \times \frac{1}{\text{Cot. } x'}$$

from which α may be easily found.

When R is nearly equal to r, as in different kinds of crown and flint glass, we have

$$x' = \text{A, and}$$

$$\text{Tang. } \overline{\alpha - \text{A}} = \frac{dR}{dr} - 1 \times \frac{1}{\text{Cot. A}}$$

The angle a, which corrects the refraction of another angle A, may be obtained from the following formulæ:

$$\text{Sin.}\, x = \frac{\text{R} \times \text{Sin. A}}{r}, \text{ and}$$

$$\text{Sin.}\, \overline{a - x} = \frac{\text{Sin. } a - \text{A}}{\text{R}}$$

In the measurement of mean refractive powers, the preceding results will be of great importance, as they enable us to ascertain, very near the truth, the colour and position of the ray which divides the spectrum formed by any particular substance. In determining the mean refractive powers of chromate of lead, realgar, and oil of cassia, which exert such a powerful influence in separating the extreme rays, this information is absolutely necessary to obtain a correct result. The refractive power of the bisecting ray cannot, by any method, be deduced from that of the extreme red and violet pencils; and therefore, in determining the mean index of refraction, every thing must depend upon the accuracy with which we judge of the mean ray. In a spectrum formed by flint glass, the extreme violet rays are sufficiently well defined, to enable the observer to determine with his unassisted eye the ray by which

it is bisected; but, in chromate of lead, realgar, and oil of cassia, which are not very transparent substances, and which produce very large, and therefore very faint spectra by small refracting angles, the extreme violet rays cease to be visible, and the eye is no longer able to determine the colour and position of the mean ray.

The following general results will be found of considerable use in the measurement of refractive powers, and particularly to those who may repeat the experiments on chromate of lead and realgar, which have been given in another part of this volume. They are founded on the supposition, that, in a spectrum formed by crown glass, the first green ray, or the ray which divides the green and blue spaces, is equidistant from the extreme red and violet. But, according to Dr Wollaston's ingenious observations on the prismatic spectrum, one of the blue rays will probably be the mean ray in a spectrum formed by crown glass.

1. In a spectrum formed by sulphuric acid, the mean ray that bisects the spectrum will be nearly about the middle of the green space.

2. In the spectrum formed by the muriatic acid, the mean ray will be about half way between the

middle of the green space and the junction of the green and blue spaces.

3. In the spectrum formed by crown glass, the mean ray will be at the confines of the green and blue spaces.

4. In a spectrum formed by flint glass, the mean ray will be advanced a little upon the blue space.

5. In a spectrum formed by oil of cassia, the mean ray will be considerably advanced upon the blue space.

6. And, reasoning from analogy, (as I have made no experiments on realgar and chromate of lead, on account of their imperfect transparency,) the mean ray in a spectrum formed by these bodies, but particularly the latter, must be very considerably advanced upon the blue space.

It appears, from the Experiments 61, 62, 63. that there is a third, or a *tertiary spectrum*, as it may be called, produced by varying the inclination of the first prism to the incident rays. As this new spectrum is formed when the two prisms are made out of the same substance, and consequently exercise the same action on the differently coloured rays, it is obviously owing to the deviation of the mean refrangible ray, which

we had before deduced from the constant ratio of the sines of incidence and refraction. This spectrum is not composed of such vivid colours as the secondary spectrum, but the fringes are sufficiently distinct, and the uncorrected green colour is always towards the vertex of the smaller prism, which is inclined so as to produce a dispersion equal to that of the large one. Hence it follows, in conformity with our former theoretical deductions, that, by increasing the refraction either by means of a large angle of the prism, or by a change in its inclination, the red and green spaces are contracted, and the blue and violet ones expanded.

This result, however, is completely the reverse of one which Dr Wollaston obtained with a prism of flint glass. In the ordinary position of his prism, the green and red spaces were to the blue and violet spaces as 39 to 61; but by altering its inclination so as to increase the dispersion of the colours, the red and green spaces were expanded to 42, while the blue and violet ones were contracted to 58, the length of the whole spectrum being supposed to be 100. We have no doubt that Dr Wollaston has measured accurately the relative magnitudes of the two halves of the spectrum under these different circumstances; but we sus-

pect that, owing to the diminution of light by increasing the dispersion, the extreme violet rays were not visible, and therefore that the blue and violet half of the spectrum appeared to be contracted, when it was in reality expanded.

The same experiments, which prove the existence of a tertiary spectrum, exhibit the no less singular phenomenon of a refraction without colour, by means of two prisms of the same substance. This effect, which has hitherto been considered as an impossible one, appears to arise from the dispersion increasing at a greater rate than the mean refraction, in consequence of altering the inclination of the incident ray; and seems to hold out the possibility of constructing an achromatic object lens, by combining two lenses of the same kind of glass.

The third series of experiments, from Exp. 66 to Exp. 69, exhibit results in which the tertiary spectrum is made to oppose and correct the secondary spectrum. In Exp. 67, the tertiary spectrum only produces a partial correction of the secondary spectrum. In Exp. 68, the secondary spectrum is completely corrected; while in Exp. 66, the secondary spectrum is more than corrected by the tertiary spectrum, the uncorrected green being to-

wards the vertex of the crown glass. These results, while they completely establish the existence of the new spectrum, seem to hold out the possibility of removing, or at least diminishing, the uncorrected colour, which is the principal obstacle to the perfection of the achromatic telescope.

The extreme minuteness of the uncorrected fringes, which form the secondary spectrum, renders it quite inpracticable to ascertain either their relative or absolute magnitude, by any of the usual methods of measuring space. The most delicate micrometer would completely fail in such an attempt, and no confidence could be placed in ruder measurements.

The existence of the tertiary spectrum, however, relieves us from this embarrassment, and furnishes us with an accurate method of ascertaining, in every case, the magnitude of the secondary spectrum. As the tertiary spectrum is not produced in virtue of any specific quality of the refracting media, but depends altogether upon the angles by which the light is refracted at the two surfaces of the prism, its magnitude is capable of being determined by direct calculation. When the tertiary spectrum, therefore, corrects the secondary spectrum, as in Exp. 68, we have only to compute the magnitude of the former, or the de-

viation of the mean refrangible ray, as produced by the crown glass; and by calculating the magnitude of the small tertiary spectrum, or the minute deviation of the mean ray which is occasioned by the flint glass prism, we shall obtain a measure of the secondary spectrum of the flint glass arising from its specific attraction for the differently coloured rays.

Having thus given a full view of the experiments which I have made on the secondary and the tertiary spectrum, I shall conclude this Chapter by pointing out the application of the results to the improvement of the achromatic telescope.

The imperfections of this instrument arise from two causes,—from the partial correction of colour, which is the consequence of an inequality in the coloured spaces of the spectra produced by crown and flint glass; and from the difficulty of procuring flint glass free of veins and specks. No attempt, so far as I know, has ever been made to remove or even to diminish the first of these evils; and the high rewards which have been offered for good flint glass, by the Board of Longitude in England, and the Academy of Sciences in France, have not yet produced any beneficial effects. The following maxims, founded upon direct ex-

periment, may probably contribute to the improvement of this valuable instrument.

1. It appears, from the preceding experiments, that the uncorrected colour diminishes in general with the difference of the dispersive powers of the two prisms or lenses by which it is produced, and in a combination of crown and flint glass, whose dispersive powers are 0.036 0.052, the outstanding colours are very considerable. The *uncorrected colour may therefore be greatly diminished, by using flint glass with a dispersive power as low as possible.* The dispersive powers of different kinds of flint glass vary from 0.045 to 0.052, and if Dr Robison's experiments have been rightly reduced, he found some so low as 0.038.* The practical optician therefore, should always select the flint glass which has the least dispersive power, and though this will render it necessary to diminish the radii of the surfaces, the secondary spectrum will be very much reduced.

2. As the crown glass likewise varies consider-

* Mr Tulley of Islington, employs flint glass between the specific gravities of 3.466 and 3.192, and as he finds that the refractive and dispersive powers increase very nearly with the density of the glass, the first of these kinds of glass must exceed the other very much in separating the extreme rays. In the glass whose density is 3.466, the ratio of refraction in the

ably in its dispersive power, according to the proportion of the ingredients of which it is composed, it would be of great importance to use that kind of crown glass which has the least power of separating the extreme rays. This will enable the optician to employ a kind of flint glass still lower in dispersive power, and thus to diminish still more the uncorrected fringes.

3. While the use of two kinds of glass that do not differ widely in dispersive power, enables us to diminish the secondary spectrum, and thus to improve considerably the achromatic telescope; it allows us at the same time to remove, in some measure, the other obstacle to the perfection of that instrument. Since a flint glass with a low dispersive power is required, it will be necessary to put only a small portion of lead into its com-

crown to that in the flint glass, is as 1 to 1.74, and the radii of curvature are

$$\begin{matrix} a=14.3 & a'=18.0 \\ b=19.4 & b'=72.0 \end{matrix} \quad F=44 \text{ inches.}$$

In the glass whose density is 3.192, the ratio of refraction is as 1 to 1.52, and the radii of curvature are

$$\begin{matrix} a=11.5 & a'=15.25 \\ b=16.8 & b'=32.50 \end{matrix} \quad F=44 \text{ inches.}$$

See the article ACHROMATIC TELESCOPE, in the *Edinburgh Encyclopædia*, 2d Edit. where I have published several forms for achromatic object glasses, which were communicated to me by that celebrated optician.

position. We may therefore expect, that the ingredients will form a more homogeneous mass when the quantity of lead is diminished, and that the glass will be less affected with those veins and specks, which render it so unfit for optical purposes.*

In order to follow out these views, I have projected a set of experiments on the composition of a proper glass for achromatic telescopes; and Sir George Mackenzie, Bart. who is more familiar than I am with such operations, has had the goodness to assist me in this important enquiry.

The experiments in pages 383 and 384, which form the second series, appear to hold out some prospect of producing achromatic refraction by two lenses of the same substance. Dollond, and every subsequent optician, would have pronounced this to be altogether impossible; but the result of the 63d Experiment shews, that it can be effected with prisms by means of an arrangement which we have represented in Fig. 5, of Plate

* As it is almost impossible to procure a piece of good flint glass more than 4 or 5 inches in diameter, might not a lens of any magnitude be composed of separate pieces of good glass from the same pot, firmly cemented together, and afterwards ground and polished? In the article BURNING INSTRUMENTS, in the *Edinburgh Encyclopædia*, vol. v. p. 143, I have proposed this construction for large burning lenses.

XII. The prism B, which has a smaller refracting angle than the prism A, has the line which bisects that angle inclined to the incident ray RR, in consequence of which, its dispersion is increased in a greater ratio than its refraction, so as to correct the dispersion of A without correcting its refraction. Hence the ray RD will emerge colourless, and will meet the axis CD.

This arrangement of the prisms may be imitated with lenses, as in Fig. 6. where the convex lens A corresponds with the prism A, as in Fig. 5. and the convex meniscus B with the prism B. We have made the lens A plano-convex, as the combined object-glass will thus produce a more perfect correction of the spherical aberration. It is probable that the best form would be that in which B is as a concave meniscus, and A a convex meniscus, having its convex side turned towards the eye-piece. I have made some experiments with glasses of this kind, but though I have observed an evident diminution of the chromatic aberration, yet owing, probably to a want of proper lenses, I have not succeeded in removing it.

In order to assist the lens B in producing a spectrum equal to that produced by A, it might be formed of another kind of crown glass, having a greater dispersive power than A.

CHAP II.

Description of a New Compound Microscope for examining objects of Natural History, and capable of being rendered Achromatic.

The construction both of single and compound microscopes has, within the last fifty years, been brought to a great degree of perfection; and for all the purposes of amusement and general observation, these instruments may be considered as sufficiently perfect. But when we employ the microscope as an instrument of discovery, to examine those phenomena of the natural world which are beyond the reach of unassisted vision, and when we use it in ascertaining the anatomical and physiological structure of plants, insects, and animalculæ, we soon find, that a limit, apparently insuperable, is set to the progress of discovery, and that it is only some of the ruder and more palpable functions of these evanescent animals that we are able to bring under observation. Na-

turalists, indeed, are less acquainted with the organization of the microscopic world, and the beings by which it is peopled, than astronomers are with those remote systems of the universe which appear in the form of nebulæ and double stars. It was the improvement of the telescope alone which enabled Dr Herschel to fix the views of astronomers upon those regions of space, to which, at a former period, their imaginations could scarcely extend; and when the microscope shall have received a similar improvement, we may look for discoveries equally interesting, though less stupendous, even in those portions of space which are daily trampled under the foot of man.

It is both important and interesting to inquire into the cause of this limitation of microscopical discovery. The construction of single lenses for the simplest form of the instrument, has been brought to great perfection. I have in my possession glasses executed by Mr Shuttleworth, of the focal length of $\frac{1}{30}$, $\frac{1}{40}$, and $\frac{1}{50}$ of an inch, which are ground with great accuracy; and the performance of single lenses has been recently improved by Dr Wollaston, who separates two hemispherical segments by means of a small plate of brass perforated in the centre. We cannot, therefore, expect any essential improvement in the single

microscope, unless from the discovery of some transparent substance, which, like the diamond, combines a high refractive power, with a low power of dispersion.

In the combination of single lenses to form the compound microscope, opticians have likewise arrived at a great degree of perfection. The aberration of refrangibility can now be completely removed by a suitable arrangement of the individual lenses; and every artifice has been exhausted in suiting the apparatus to the various tastes of the purchasers, and to every purpose of popular observation.

No attempt, however, appears to have been made by opticians to fit up the microscope as an instrument of discovery, to second the labours of the naturalist in preparing the subjects of his research, and to *accommodate the instrument to that particular kind of preparation which is indispensibly necessary for the preservation and inspection of minute objects.*

In perusing the writings of those naturalists who have applied the microscope to the examination of minute objects, we find, that the most difficult and perplexing part of their labour consisted in preserving and preparing the different insects and substances which they wished to in-

spect. Small insects instantly shrivel up and lose their natural form as soon as they are killed, and the minute parts of plants suffer a similar change from exposure to the air. Hence Swammerdam and Lyonet killed the insects which they meant to examine, by suffocating them either in water, spirit of turpentine, or diluted spirits of wine. The softness and transparency of their parts were thus preserved during the process of dissection, and when they were completely developed, the insect was allowed to dry before it was presented to the microscope. Its parts were consequently contracted, and lost not only their proper shape, but that plumpness, and that freshness of colour which they possessed when alive.

In the preparation, indeed, of almost every object of natural history that is composed of minute and delicate parts, it must be preserved by immersion in a fluid; the dissection must often be performed in the same medium; it must be freed from all adhesive and extraneous substances, by maceration and ablution in water; and when it has undergone these operations, it is in a state of perfection for the microscope. Every subsequent change which it undergoes is highly injurious: it shrivels and collapses by being dried; its natural polish and brilliancy are impaired; the mi-

nute parts, such as the hairs and down, adhere to one another, and the general form of the object, as well as the disposition of its individual parts, can no longer be distinctly seen.

It is therefore a matter of considerable importance to be able to examine the object when wet, and before it has suffered any of these changes; and by fitting up the microscope in the following manner, this may be effected without even exposing the object to the air.

The object glass of the compound microscope should have the radius of the immersed surface about nine times the focal distance of the lens, and the side next the eye, about three fifths of the same distance. This lens should be fixed into its tube with a cement which will resist the action of water or spirits of wine; and the tube, or the part of it which holds the lens, should have an universal motion, so that the axis of the lens may coincide to the utmost exactness with the axis of the tubes which contain the other glasses.

Several small glass vessels must then be provided, having different depths, from one inch to three inches, and having their bottom composed of a piece of flat glass, for the purpose of admitting freely the reflected light which is intended to illuminate the object. The fluid in which the ob-

ject has been preserved, or prepared, is next put into the vessel; and the object itself, placed upon a glass stage, or if necessary fixed to it, is immersed in the fluid. The glass vessel is now laid upon the arm of the microscope, which usually holds the object, and the lens is brought into contact with the fluid in the vessel. The rays which diverge from the object emerge directly from the fluid into the object glass, and therefore suffer a less refraction than if it had been made from air; but the focal length of the lens is very little increased, on account of the great radius of its anterior surface. The object may now be observed with perfect distinctness, unaffected by any agitation of the fluid;—its parts will be seen in their finest state of preservation;—delicate muscular fibres, and the hairs and down upon insects, will be kept separate by the buoyancy of the fluid; and if the object when alive, or in its most perfect state, had a smooth surface, its natural polish will not only be preserved but heightened by contact with the fluid. Aquatic plants and animals will thus be seen with unusual distinctness, and shells and unpolished minerals will have a brilliancy communicated to their surfaces which they could never have received from the hands of the lapidary. If the specific gravity

of the substance under examination should happen to be less than that of the fluid, and if it cannot easily be fixed to the glass stage, it may be kept from rising to the top by a piece of thin parallel glass, or by a small grating of silver wire stretched across the vessel.

In order to find the magnifying power of such a microscope, let

f = Focal length of object-glass.
F = Focal length of amplifying glass.
d = Distance of object from object-glass.
D = Distance between the object-glass and the amplifying-glass.
m = Magnifying power of the eye-glass.
M = Magnifying power of the microscope.

Then, from the reasoning in p. 64, 65, we shall find

$$x = \frac{df}{d-f} - \mathrm{D}, \text{ and}$$

$$\mathrm{M} = \frac{\mathrm{F}}{x+\mathrm{F}} \times \frac{f}{d-f} \times m$$

When the object-glass is equally convex, and when its anterior surface is immersed in water, we shall have

$$x = \frac{1.37\,fd}{d-1.37f}, \text{ and}$$

$$\mathrm{M} = \frac{\mathrm{F}}{x+\mathrm{F}} \times \frac{1.37\,f}{d-1.37f} \times m$$

When the object-glass is unequally convex, and

a, b are its radii, a being immersed in water, we shall have $f = \frac{5ab}{2.65a + b}$; and by using the value of f in the first equations, the magnifying power will be found.

The method of fitting up and using the compound microscope, which has now been described, enables us, in a very simple manner, to render the object-glass perfectly achromatic, without the assistance of any additional lens. The rays which proceed from the object immersed in the fluid, will form an image of it nearly at the same distance behind the lens, as if the object had been placed in air, and the rays transmitted through a plain concave lens of the fluid combined with the object-glass. If we, therefore, employ a fluid whose dispersive power exceeds that of the object-glass, and accommodate the radius of the anterior surface of that lens to the difference of their dispersive powers, the image will be formed perfectly free from any of the primary colours of the spectrum. The fluids most proper for this purpose are,

Oil of cassia.	Oil of sassafras.
Oil of anise seeds.	Oil of sweet fennel seeds.
Oil of cummin.	Oil of spearmint.
Oil of cloves.	Oil of pimento.

These oils are arranged in the order of their dispersive powers; and when those at the top of the list are used, the anterior surface of the object-glass will require a greater radius of curvature than when those at the bottom of the list are employed. Thus, in order to render the object-glass achromatic, when it is made of crown glass, and when the fluid is oil of cassia, the radius of the anterior or immersed surface, should be to that of the surface next the eye as 2.5 to 1. Lest these proportions should not exactly correct the chromatic aberration, it would be preferable to make the radii as 2.2 to 1, and then reduce the dispersive power of the oil of cassia by oil of olives, or any other less dispersive oil, till the correction of colour is complete. If the oil of sweet fennel seeds is used, the radius of the anterior should be to that of the posterior surface, as 0.8 to 1.

CHAP. III.

Description of a New Solar Microscope, which can be rendered Achromatic.

THE principles upon which the instrument described in the preceding Chapter is constructed, may be applied with peculiar advantage to the solar microscope, whether it is employed for the examination of opaque or transparent objects.

The method of fitting up the solar microscope, to render it susceptible of this improvement, is represented in Plate XII. Fig. 7. where AB is the illuminating lens which receives the parallel rays of the sun, and throws them upon the object. The object lens CD is firmly cemented into one end of a tube *m* C D *n*, which has a tubular opening at E; and at the other end of the tube is cemented a circular piece of parallel glass *m n*. The tube *m* C D *n* is then filled with water, or any other fluid; and the object, when fixed upon a slider, or held with a pair of forceps, is introdu-

ced into the fluid at the opening E. The slider, or the forceps, may be easily rendered moveable, so that the object may be placed at a proper distance from B; or the adjustment may be effected by a motion of the screen on which the image is projected. The plate of glass mn might be removed, and the whole of the space between AB and CD filled with fluid; but if the fluid had any tinge of colour, the transmitted light would, in this case, partake of it, and injure the distinctness of the image.

If the microscope is fitted up for the examination of transparent bodies, it is obvious, that the image will be much more perfect than if it had been formed in the common way. The opacity which arises from a contraction of parts is thus completely removed, and an additional transparency is communicated by the fluid, which could not have been obtained in any other way. Substances, indeed, which with the common solar microscope appear opaque, will, in the present form of the instrument, exhibit a very great degree of transparency. The advantages arising from immersion in a fluid, which have been very fully stated in the preceding Chapter, apply with peculiar force when the objects are used in the solar microscope.

This microscope may be rendered perfectly achromatic, by using the same fluids, and by giving the lens nearly the same radii of curvature, which have been mentioned in the preceding Chapter.

CHAP. IV.

Description of New Fluid Microscopes.

THE first idea of fluid microscopes was suggested by Mr Stephen Gray, who published an account of them in the *Transactions of the Royal Society*.* They consisted merely of a drop of water, which was taken up on the point of a pin, and placed in a small hole one-thirtieth of an inch in diameter, in the middle of a spherical cavity about one-eighth of an inch broad, and a little deeper than half the thickness of the plate. On the opposite side of the plate was another spherical cavity, half as broad as the former, and so deep, as to reduce the circumference of the small hole to a sharp edge. When the water is placed in these cavities, it will form a double convex lens with unequal convexities, which may be employed like any other single microscope in the examination of minute objects.

* See the *Philosophical Transactions*, No. 221, 223, and Smith's *Optics*, vol. ii. p. 394.

As water, however, has a considerable dispersive power, and a low power of refraction, fluid microscopes of a more perfect kind may be formed, by using sulphuric acid, castor oil, oil of ambergrease, or alcohol. The sulphuric acid has a very low dispersive power, and a greater refractive power than water, and will, therefore, make a more perfect lens than any other fluid body. Castor oil may be employed with almost equal advantage; and oil of ambergrease and alcohol would answer the same purpose from their optical properties, though their volatility may render them less easily managed.

The best method, however, of constructing fluid microscopes, is to take Canada balsam, balsam of capivi, or pure turpentine varnish, and let a drop of any of them fall upon a thin piece of parallel glass. The drop will form a plano-convex lens, and its focal length may be regulated by the quantity of fluid which is used. These fluid lenses are represented in Fig. 8. of Plate XII. as suspended upon the parallel glass; but the proper position is when the plate of glass is horizontal. If the lens is uppermost, the gravity of the fluid will make it more flat, and diminish its focal length: This, however, may be avoided, and the contrary effect produced, by inverting the piece of glass. If

these lenses are preserved from dust, they will be as durable as those which are made of glass; and when thick Canada balsam is used, the lenses will soon be indurated into a hard gummy substance, and resist any change of figure from the gravitation of their parts.

I have sometimes employed these fluid lenses as the object glasses of compound microscopes; and I even constructed a good compound microscope, in which both the object glass and the eye glass were made of a fluid.

In the present improved state of optical instruments, these microscopes cannot be of any essential service; but occasions sometimes occur for microscopical observations when lenses are not to be had, and when the materials of a fluid microscope are the only substitute which the observer can command.

CHAP. V.

Description of adjusting Microscopes for seeing at two different Distances at the same time.

There are numerous observations, both of a mechanical and a physical nature, in which it is of the greatest consequence to adjust the eye to objects situated at different distances. In measuring the diameters of minute objects, by observing the space which they occupy upon a distant plane surface, it is requisite, to the accuracy of the result, that both the object and the plane be seen distinctly at the same time. In the superior and inferior adjustments of the barometer, and in every case where it is required to place the axis of the eye either parallel to, or coincident with, a given line, the same kind of adjustment is of essential importance.

When the more remote of the two points which are to be seen at the same time, is at a greater dis-

tance than seven or eight inches, the shortest limit of distinct vision, the adjusting microscope may be formed by drilling a small hole through the centre of the lens, as in Fig. 9. Plate XII.; or, what may sometimes be more convenient, by cementing upon the centre of each surface of the lens two small circular pieces of glass *m*, *n*, Fig. 10, by a fluid such as Canada balsam, which has nearly the same refractive power with the glass. The central part of the lens will thus be reduced to a plane surface, and will have the same effect as the form exhibited in Fig. 9. When the rays, therefore, from the more distant point, pass through the perforation *o*, or through the planes of glass *m*, *n*, and fall upon the central part of the pupil, they will form a distinct image of it upon the retina, while the rays from the nearer point passing through the lens, will be incident upon the outer portion of the pupil, and form an equally distinct image of it upon the retina. The coincidence of the one point with the other, or the space which one of them occupies upon any plane, may thus be distinctly observed.

If the distance of both the points is less than seven or eight inches, the adjusting microscope should be constructed as in Fig. 11, where a plane of glass is cemented to the lens by a small

circular portion of Canada balsam, or any other viscid fluid, which has such a refractive power that the increased focal length of the central part of the lens may be to its real focal length, as the distance of the remoter point is to the distance of the nearer point. The remoter point will then be seen distinctly through the central portion of the lens, while the nearer point will be seen with equal distinctness through the outer portion. The same effect will be produced by the construction in Fig. 12, where the cement forms a ring at the circumference of the lens.

In order to see three points at the same instant with perfect distinctness, which may sometimes be necessary, we must adopt the construction in Fig. 13, where a very small circular plane of glass is cemented by Canada balsam to the anterior surface of the lens, and a small circular ring of glass cemented, on the opposite side, to the portion of the lens immediately surrounding the first circular plane. The lens will thus be divided into three zones, having three different focal lengths, which may be varied in any required proportion, by altering the radii of the two surfaces, or changing the refractive power of the cement. In order to avoid any loss of light arising from the circumferences of the glass planes not being transparent,

each of them may be extended to the very circumference of the lens. In all the constructions which have been mentioned, the different apertures must be accommodated with great care to the size of the pupil, which will, of course, vary with the intensity of the light in which the observations are made.

CHAP. VI.

Description of Opera Glasses and Night Glasses, upon a New Construction.

After Dollond had constructed the achromatic telescope, the theory of that instrument was viewed, in almost every possible aspect, by Euler, Clairaut, D'Alembert, and Boscovich. D'Alembert, in particular, has shewn, that an achromatic telescope may be constructed with a single object glass, and with a single eye glass of a different refractive and dispersive power. In this form of the instrument, the eye glass must be concave; and the glass, of which it is composed, must exceed in dispersive power the glass from which the object lens is made. This construction of the telescope, however, was abandoned as soon as it was suggested. The substances which were then known to differ most in dispersive power, were crown and flint glass; and in order to have a perfect correction of colour with

these substances, the *telescope could not magnify more than one and a third times.* An instrument of this kind, which can scarcely be said to have any magnifying power at all, is of no use whatever; and I believe that no artist has attempted to construct it.

The discovery, however, of the enormous dispersive power of oil of cassia, and some other essential oils, which I have already noticed in another part of this volume, renders such a form of the instrument no longer impracticable; and though we can never obtain a magnifying power sufficient for astronomical purposes, yet opera glasses and small telescopes may thus be made with wonderful precision.

If an opera glass were constructed with an object lens perfectly achromatic, and made of crown and flint glass, the aberration of colour produced by the eye glass would still remain, as there is not sufficient space for that combination of single lenses, which would at the same time erect the object and remove the uncorrected colour.

The construction, therefore, which has been mentioned, is peculiarly fitted for opera glasses, as the dispersion of the object glass is completely removed by the opposite dispersion of the eye glass.

If we suppose the object glass to be equally convex on both sides, and the eye glass equally concave, and make

$R =$ index of refraction for the object glass.
$r =$ index of refraction for the eye glass.
$dR =$ Part of the whole refraction to which the dispersion is equal, in the object-glass.
$dr =$ Part of the whole refraction to which the dispersion is equal, in the eye-glass.
$F =$ Focal length of the object glass.
$f =$ Focal length of the eye glass.
$A =$ Radius of the object glass.
$a =$ Radius of the eye glass.

Then it may easily be shewn, that the concave eye glass, will exactly correct the dispersion of the convex object-glass, when

$$A : a = \frac{dr}{(r-1)^2} : \frac{dR}{(R-1)^2}$$

The application of this formula to various combinations, will point out the substances which are most fit to be employed in instruments of this kind.

I. Object Glass, *Crown Glass;* Eye Glass, *Flint-glass.*

In this case we have the following values: *

* See the Tables of Refractive and Dispersive Powers, p. 279. and 315.

$$\begin{aligned} R &= 1.544 \\ r &= 1.616 \\ dR &= 0.020 \\ dr &= 0.032 \text{ and} \end{aligned}$$

$$A : a = \frac{0.032}{0.379} : \frac{0.020}{0.296} = 1.25 : 1$$

Hence it follows, that in this combination the magnifying power of the instrument cannot much exeeed $1\frac{1}{4}$ times. †

II. Object Glass, *Crown Glass;* Eye Glass, *Oil of Cassia.*

In this case we have the following values:

$$\begin{aligned} R &= 1.544 \\ r &= 1.641 \\ dR &= 0.020 \\ dr &= 0.089 \text{ and} \end{aligned}$$

$$A : a = \frac{0.089}{0.411} : \frac{0.020}{0.296} = 3.24 : 1$$

Hence it follows, that in this combination the magnifying power of the instrument may be fully 3 times.

III. Object Glass, *Water;* Eye Glass, *Oil of Cassia.*

In this case we have the following values:

† The exact magnifying power may be deduced from the formulæ

$$F = \frac{A}{2R-2} \text{ and } f = \frac{a}{2r-2}$$

$$R = 1.336$$
$$r = 1.641$$
$$dR = 0.012$$
$$dr = 0.089 \text{ and}$$
$$A : a = \frac{0.089}{0.411} : \frac{0.012}{0.113} : 2.05 : 1$$

Hence it follows, that in a combination of water and oil of cassia, the magnifying power of the instrument will only be about 2 times.

IV. Object Glass, *Rock Crystal;* Eye Glass, *Oil of Cassia.*

In this form we have the following values:

$$R = 1.562$$
$$r = 1.641$$
$$dR = 0.014$$
$$dr = 0.089 \text{ and}$$
$$A : a = \frac{0.089}{0.411} : \frac{0.014}{0.316} = 5.0 : 1.$$

Hence it follows, that with this combination the magnifying power of the instrument may be nearly 6 times.

As this combination gives a greater magnifying power than any other, I have computed the following Table, which shews the length of the opera glass, the radius of each surface of the object glass, and the radius of each surface of the eye glass.

TABLE *for Opera Glasses and Small Telescopes.*

Length of the Opera Glass, or Small Telescope.	Radius of each Surface of the Object Glass.	Radius of each Surface of the Eye Glass.	Magnifying Power.
In. Dec.	Inches.	In. Dec.	
3.98	5	1.02	
4.78	6	1.22	
5.58	7	1.42	
6.38	8	1.62	
7.17	9	1.83	5.6 times.
7.97	10	2.03	
8.77	11	2.23	
9.57	12	2.43	
10.36	13	2.64	
11.16	14	2.84	

V. OBJECT GLASS, *Rock Crystal;* EYE GLASS, *Flint Glass.*

In this case we have the following values:

$$R = 1.562$$
$$r = 1.616$$
$$d\,R = 0.014$$
$$d\,r = 0.032, \text{ and}$$
$$A : a = \frac{0.032}{0.379} : \frac{0.014}{0.316} = 1.91 : 1.$$

In this combination, therefore, the instrument may have a magnifying power of nearly 2 times, which is perhaps enough in a common opera glass.

VI. Object Glass, *Rock Crystal;* Eye Glass, *Oil of Anise Seeds.*

In this case we have the following values:

$$R = 1.562$$
$$r = 1.601$$
$$dR = 0.014$$
$$dr = 0.044 \text{ and}$$
$$A : a = \frac{0.044}{0.361} : \frac{0.014}{0.316} = 2.82 : 1.$$

With this combination, the magnifying power of the instrument may be nearly 3 times.

By employing the following substances, achromatic combinations may be formed by means of single lenses, which will give a sufficient magnifying power for opera glasses.

Substances fit for Eye Glasses.	Substances fit for Object Glasses.
Glass of lead	Crown glass.
Oil of cassia.	Plate glass.
Oil of anise seeds.	Water.
Oil of cummin.	Alcohol.
Oil of cloves.	Sulphuric acid.
Oil of sassafras.	Oil of ambergrease.
Oil of sweet fennel seeds.	Rock crystal.
Oil of spearmint.	Topaz.
Oil of pimento.	

In the construction of night glasses, where much light and only a small magnifying power are

necessary, and in combinations of lenses for microscopes, the preceding principles may be successfully adopted.

When a higher magnifying power is required, than is compatible with a complete correction of the aberration of refrangibility, the greater portion of the colour may still be removed by the combinations which we have pointed out; and in cases where the fluid lenses may not be reckoned convenient, the eye glass should always be formed of a higher dispersive substance than that which is employed for the object glass.

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PLATE 1.

Fig. 1.

Fig. 2.

Fig. 5.

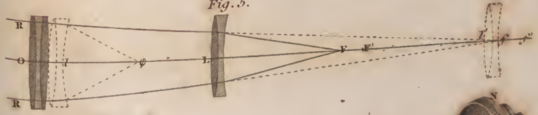


Fig. 4.

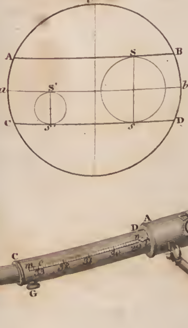
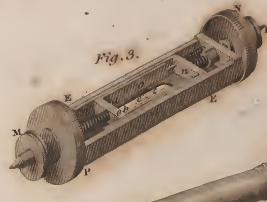


Fig. 3.



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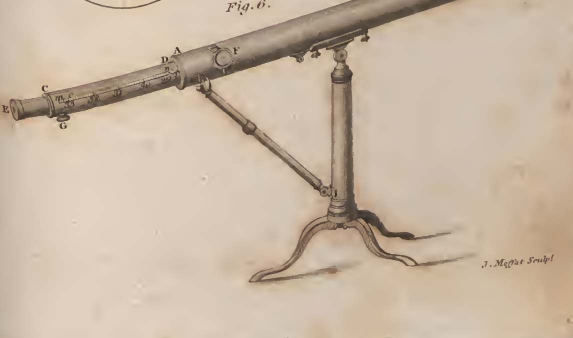

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PLATE II

Fig. 1.

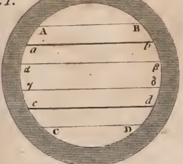


Fig. 6.

Fig. 2.

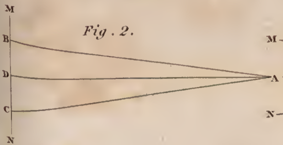
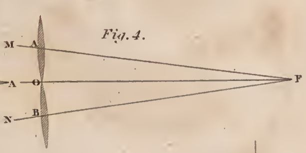


Fig. 4.



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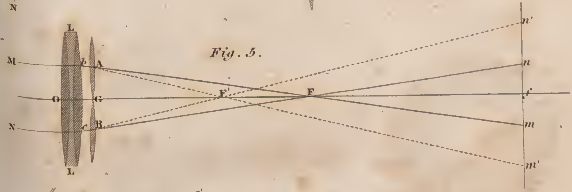
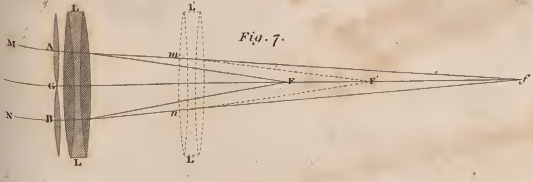


Fig. 7.



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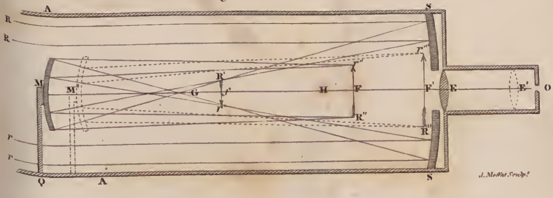

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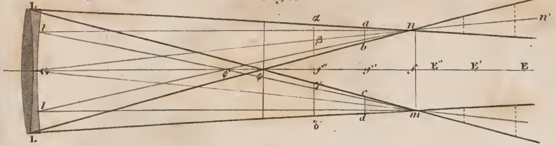
PLATE III.

Fig. 1.



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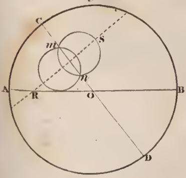

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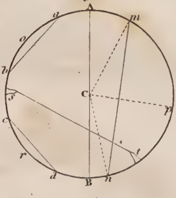

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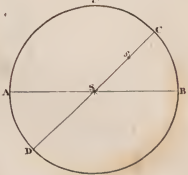

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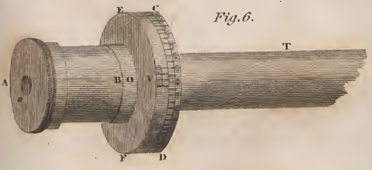
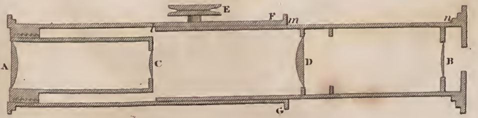


Fig. 4.

Fig. 7.



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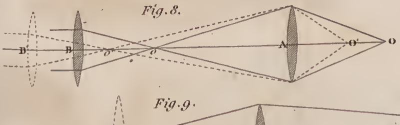

Fig. 9.

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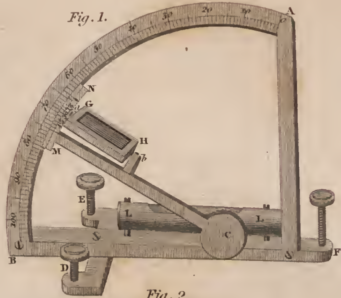
PLATE IV.

Fig. 1.



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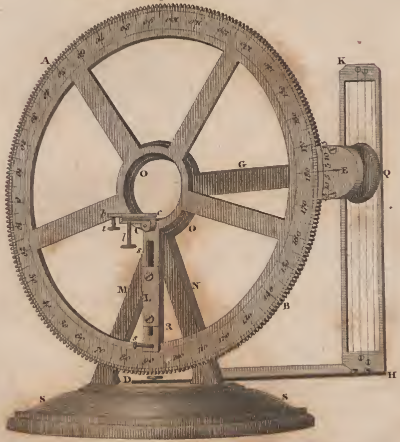

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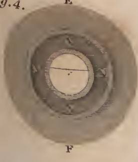

Fig. 3.

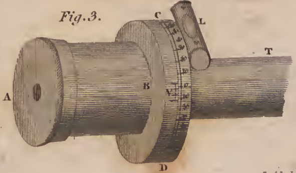

Fig. 5.

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PLATE V.

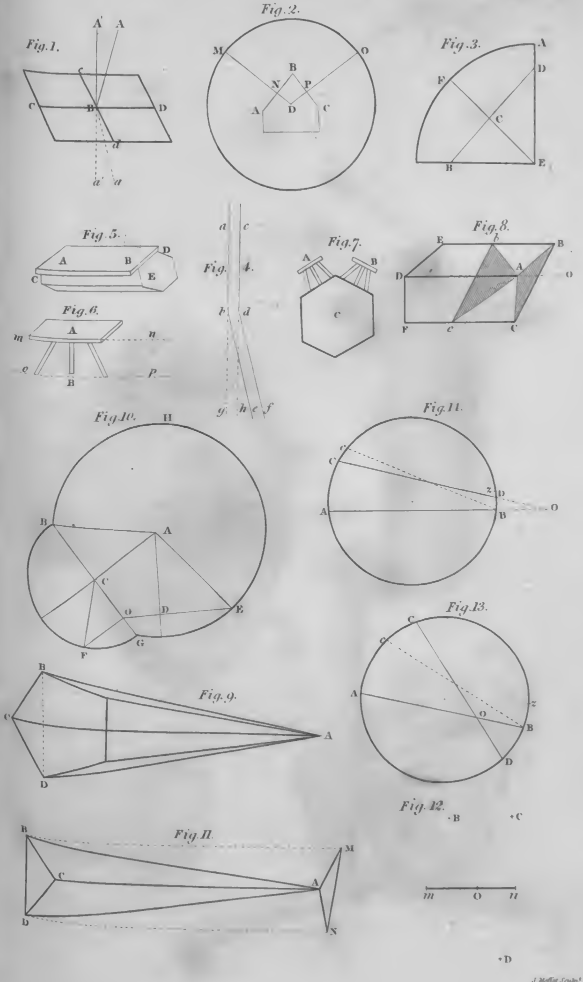
Fig.1.
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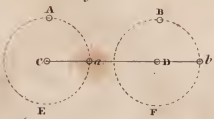
Fig.13.

PLATE VI.

Fig. 4.



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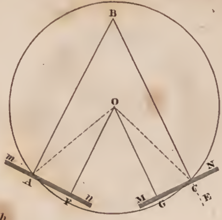

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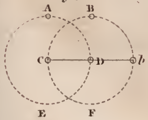

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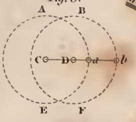

Fig. 3.

Fig. 1.

Fig. 7.

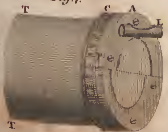

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PLATE VII.

Fig. 1.

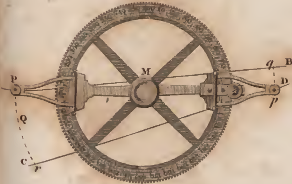
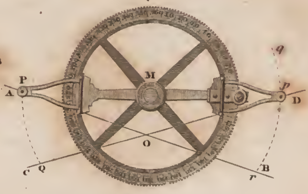


Fig. 2.



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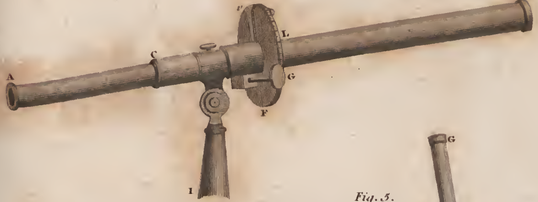

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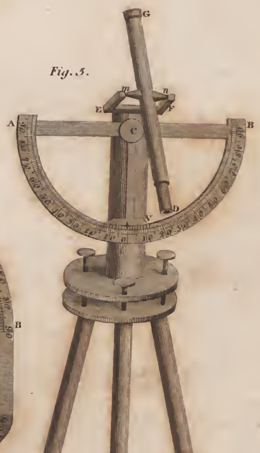

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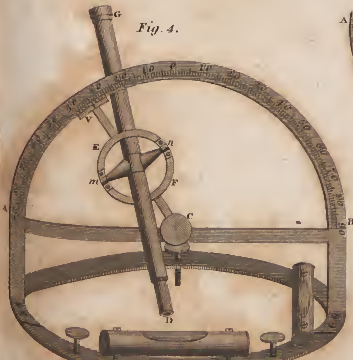

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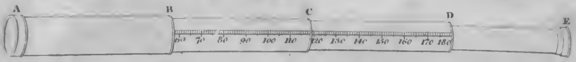
PLATE VIII.

Fig. 1



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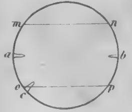
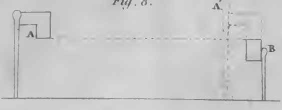


Fig. 8.



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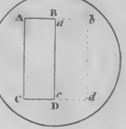

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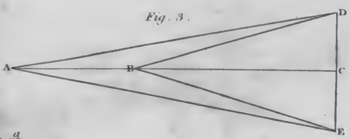

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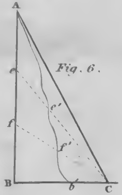

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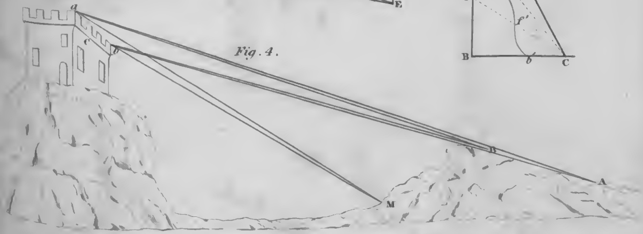

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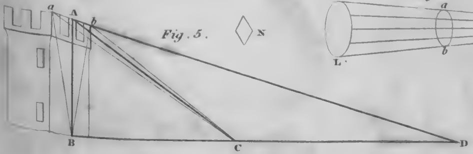

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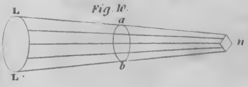

Fig. 9.

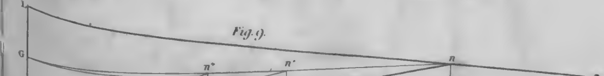

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PLATE IX.

Fig. 2.

Fig. 5.

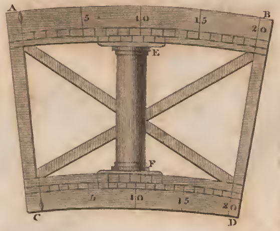


Fig. 1.

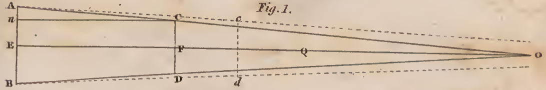


Fig. 3.

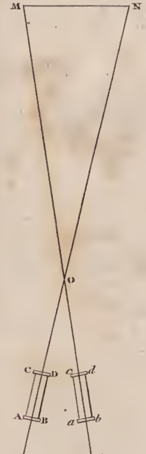
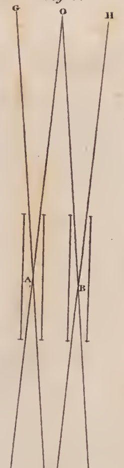


Fig. 6.



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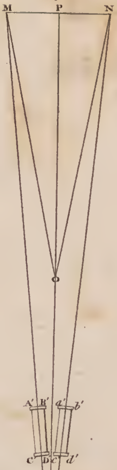

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PLATE X.

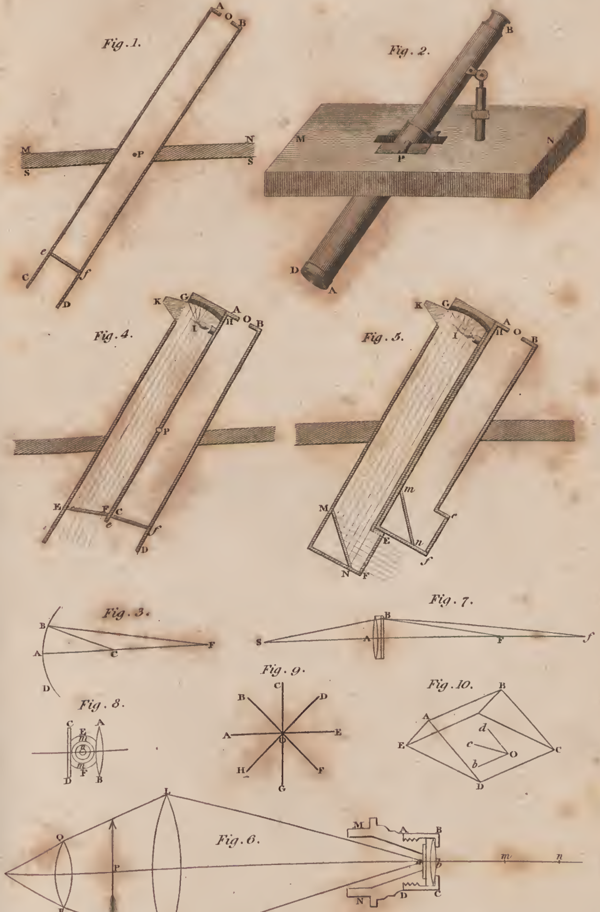
Fig. 1.
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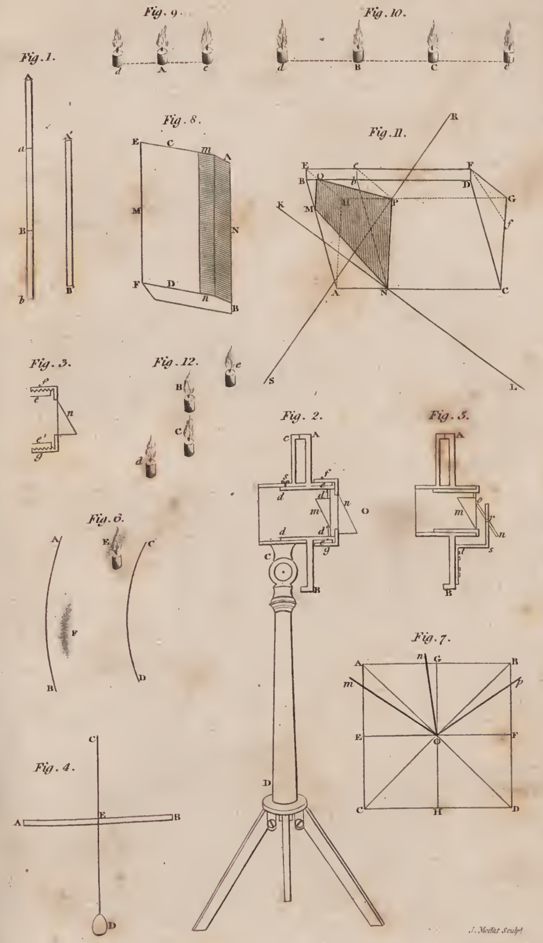
J. Moffat Sculpt

PLATE XI.



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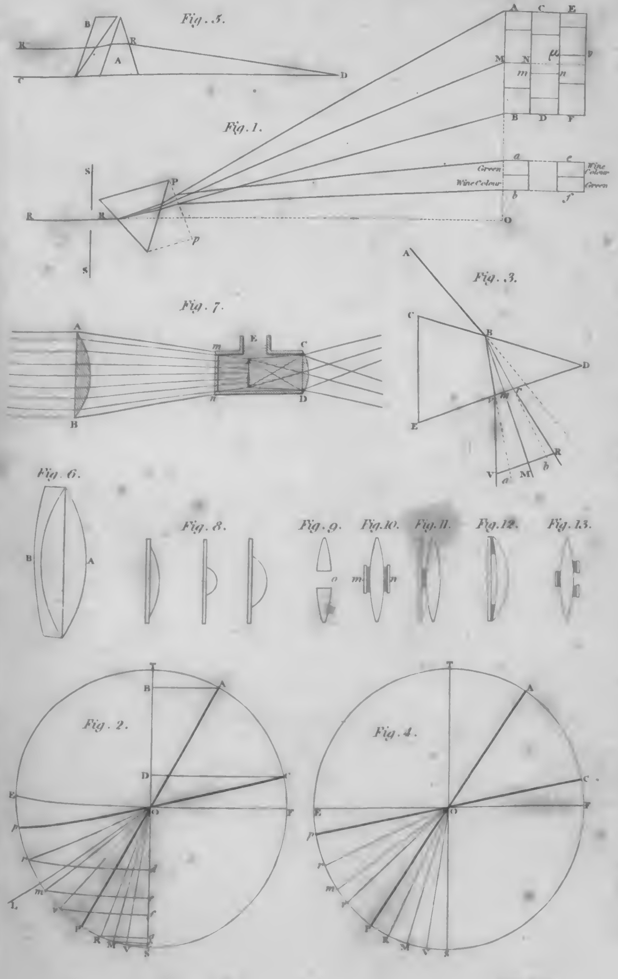
Fig. 5.
Fig. 1.
Green
Wine Colour
Wine Colour
Green
Fig. 3.
Fig. 7.
Fig. 6.
Fig. 8.
Fig. 9.
Fig. 10.
Fig. 11.
Fig. 12.
Fig. 13.
Fig. 2.
Fig. 4.